ANALYSIS AND DESIGN OF SINUSOIDAL PARAMETER PERTURBATION ADAPTIVE CONTROL SYSTEMS

YONG CHAE SON

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by

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Submitted in partial fulfillment of
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ABSTRACT

A single dimensional sinusoidal parameter perturbation adaptive control system is studied. The operation and signals at every point of the system are analyzed theoretically. Then, the system is simulated on analog computer starting with the simplest circuit and adding more components until a complete system is formed. The results of simulation study are compared with the theoretical analysis. Both agree in most cases. Finally, an analytical design of the system is attempted based on the simulation study and using a linearizing technique. It was simulated on analog computer and the performance was compared with the one predicted analytically. It does not follow exactly as predicted but the deviation is not so great. This method is useful in obtaining preliminary design which will be refined by experimental verifications.

The writer wishes to express his appreciation for the assistance and encouragement given him by Professor R. C. Dorf of the U. S. Naval Postgraduate School in this investigation.

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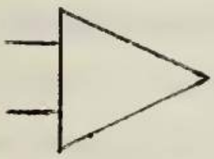
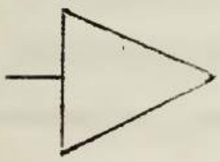
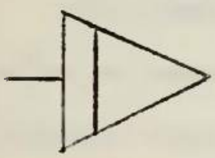
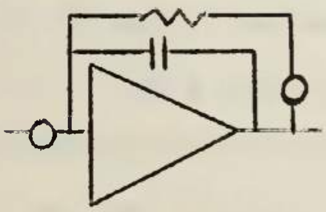
TABLE OF SYMBOLS AND ABBREVIATIONS

Symbol	Description
a_1, a_2	Constants for IP expression
A_1, $A_2 \ldots A_n$	
A_m, A_p, A_d	Attenuation factors
B	Attenuation factor of $G_3(s)$
C_1	Maximum amplitude of perturbation signal
C_p	Parameter perturbation signal
C_{y5}	Carrier signal in Y_5
e	Error
f	Frequency
$G_1(s)$, $G_2(s) \ldots G_n(s)$	
$G_m(s)$, $G_{mod}(s)$,	
$G_p(s)$,	Transfer functions of components
$G_o(s)$	Open loop transfer function of un-compensated system
$G_{oc}(s)$	Open loop transfer function of compensated system
G_{M1}	Equivalent transfer function of squarer
G_{M2}	Equivalent transfer function of detector
$G_{1equiv}(s)$	Equivalent transfer function of band pass filter
IP.	Index of performance
K_v	Static error coefficient
K_c	Critical gain
K_3, K_4, K_m	Gain constants
M_1	Squarer
M_2	Detector
M_{pt}	Maximum peak overshoot

Table of Symbols and Abbreviations

Symbol	Description
$r_1, r_2 \ldots r_n$	Roots of a characteristic equation
R	Plant input
S	Laplace variable
X	Parameter value
X_o	Optimum parameter value
X_1	Maximum amplitude of parameter drift
$Y_1, Y_2 \ldots Y_n$	Signals at each point in an circuit
ζ	Damping factor
$\theta, \theta_1, \theta_m, \theta_p$	Phase angles
$\tau_1, \tau_2, \tau_3, \tau_m$	Time constant
ω_1	Parameter perturbation frequency
ω_m	Parameter drift frequency
$\omega_h; \omega_{h1}; \omega_{h2}$	Half power frequencies of band pass filter
	Summer
	Amplifier or sign changer
	Integrator
	Lag network

1. Introduction

The adaptive control system has received much attention recently. The principle of adaptive control is to maintain a system always in an optimum operating condition whatever the affecting situation may be. There are two kinds of adaptive control systems; the signal adaptive system and the system adaptive system. The signal adaptive system is a system which can analyze incoming signals and automatically adjust the system to give an optimum output. The system adaptive system is a system which can measure its own characteristics and then change its structure or design so as to continuously maintain a desired performance. This paper deals with only the latter. The system adaptive system should have the following two essential features to achieve its goal;

(1) The measurement of its own characteristics.

(2) The generation of proper corrective signal for the controlled system according to the measurement. In the measurement of the characteristics of the system, the optimum operating condition or figure of merit is defined. The figure of merit or index of performance (IP.) is an indication of the characteristic or quality of a system. The IP may be a physically meaningful value such as heat produced per gallon of fuel or theoretical quantity such as RMS error or mean squared error.

The sinusoidal parameter perturbation adaptive control system is a method of system adaptive control and in this paper the operation and signals at every point of this system, are analyzed theoretically. Then, the system is simulated on analog computer starting from the simplest circuit. Adding more components one by one, the effect of each component is observed until the complete system is formed. The results of these simulation study are compared with the theoretical analysis. Both agree in most

cases. Finally, a design of the system is attempted analytically based on the simulation study and using a linearizing technique. It was also simulated on the analog and the digital computer and the actual performance of the system is compared with the predicted performance from the analytical design. The results show that the actual performance does not exactly agree with the performance predicted analytically but it follows reasonably close pattern predicted analytically. This is expected because the analytical method involves approximation. This method is useful to obtain a preliminary design which will be refined by experimental verification and gives a direction of improving the first trial design.

2. The Sinusoidal Parameter Perturbation Adaptive Control System.

A single dimensional sinusoidal perturbation system is as Fig. 1. The model is an optimum operating model. When the plant is operating exactly as the model there will be no error output. But when a parameter in the plant is changed to some other from the optimum parameter value due to environment change, there will be an error output. By squaring the error we get a non-negative function of error. As an index of performance we may adopt average of error squared as:

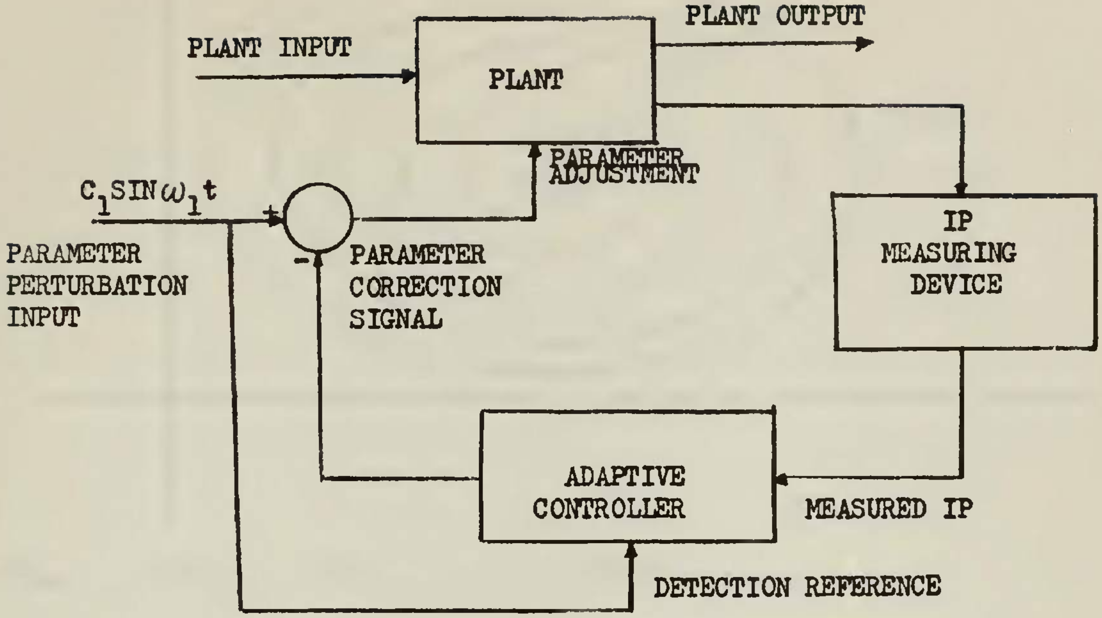
$$IP = \frac{1}{T}\int_0^T e^2 dt \qquad (2\text{-}1)$$

This quantity is a function of the characteristics of input to the plant and also of the characteristic of plant parameter drift caused by environment change. Fig. 2 shows the contours of IP(X,Y) for two dimensional case where IP is a function of two parameters X and Y. The minimum IP occurs at $X=X_0$ and $Y=Y_0$. If we fix the value of Y at Y_0 and vary only X, the IP variation with X becomes as Fig. 3, the single dimensional case. Generally the IP with one parameter variable tends to be a parabolic function of the parameter and can be approximated near the optimum parameter value as;

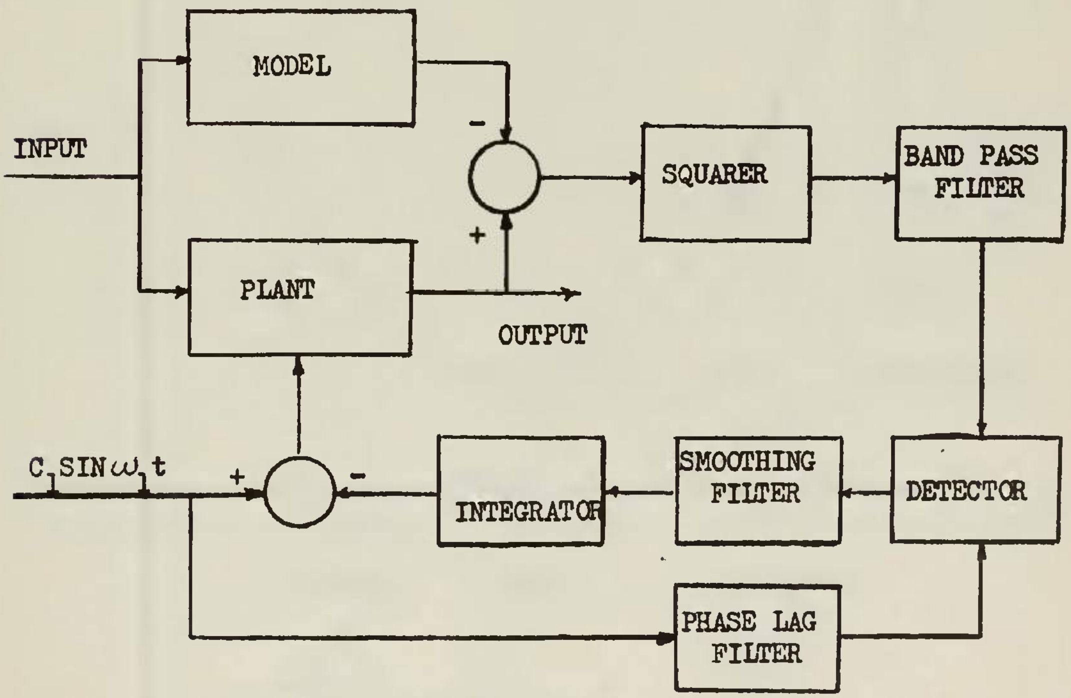
$$IP(X) = a_1(X-X_0)^2 + a_2 \qquad (2\text{-}2)$$

where X_0 is optimum parameter value and a_1 and a_2 are arbitrary constants which change the shape of IP curve such that it represents closely the actual IP variation of any plant.

In some plants a parameter has a slight or no change with respect to the environment change and such parameter is fixed at the optimum value and the



(a) General set-up



(b) Model Reference System

Fig. 1. Single Dimensional Sinusoidal Parameter Perturbation Adaptive Control System.

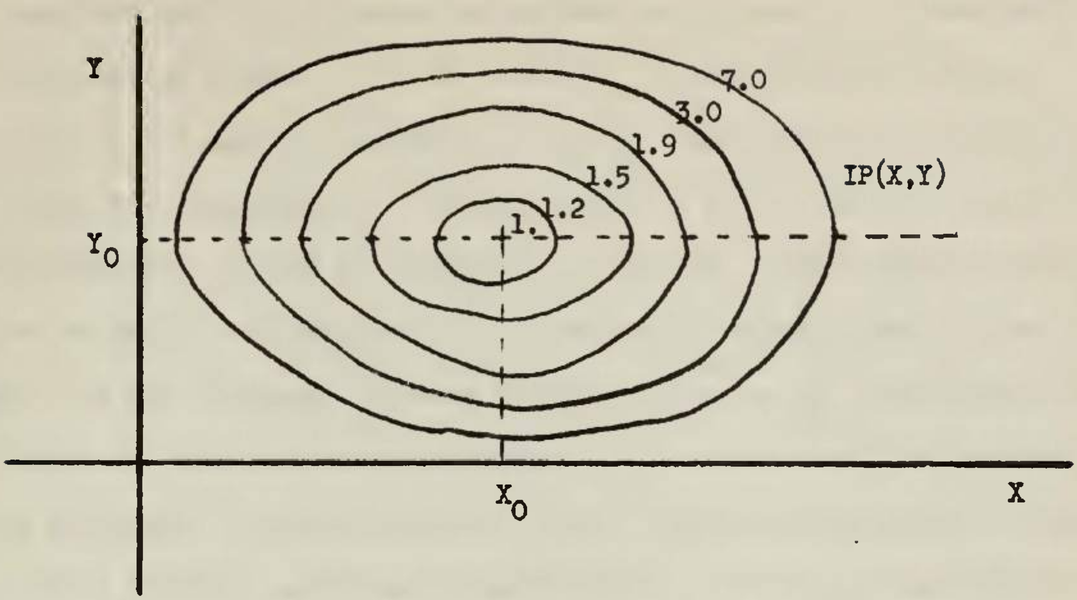


Fig. 2. Contours of IP(X,Y) for two dimensional case.

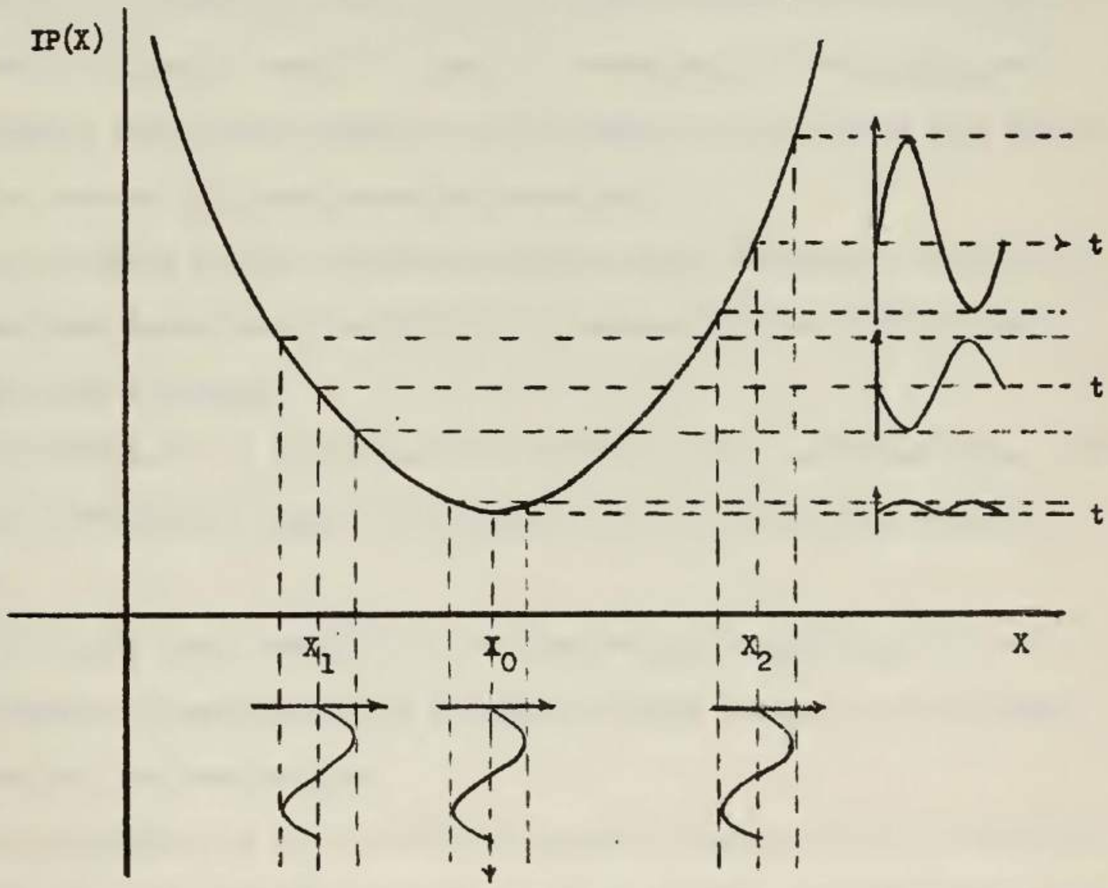


Fig. 3. IP v.s. parameter X

number of parameters to be controlled is reduced.

Referring to Fig. 3, suppose we perturb the parameter X sinusoidally by a perturbation signal, $C_p = C_1 \sin \omega_1 t$. If the actual plant parameter X is at the optimum parameter value, X_0, there will be no IP variation output at frequency ω_1. If the X value is lower than X_0, there will be an IP variation output at frequency ω_1 and the output signal will be 180° out of phase with respect to the input perturbation signal as seen in the Fig., on the contrary, if the X is higher than the X_0, the output is in phase with the input perturbation signal. It is also noted that the amplitude of the output is proprotional to $|X-X_0|$, therefore the output is considered to be a carrier, modulated by the variation of $|X-X_0|$ through the IP characteristic. The frequency of carrier is ω_1 and it carries the information about the polarity of the $X-X_0$ by its phase difference with the input perturbation signal. When this output is demodulated by multipling with the original perturbation signal, it will produce a D.C. signal with proper sense to correct the plant parameter deviation.

The bandpass filter eliminates unwanted signal components contained in the modulated signal and also affects the response of the adaptive loop to a step change of signal.

The detector is a multiplier which multiplies the modulated signal with original perturbation signal and produces positive or negative correction signal.

The filter after the detector is a smoothing filter which filters out high frequency components in the correction signal and acts as compensator to stabilize the adaptive loop.

The integrator is a D.C. motor in practical operation and it integrates correction signal and drives the plant parameter control to the optimum value.

If we want to make more than one parameter adaptive, separate loops for each different parameter must be established and use different perturbation frequency for each parameter. The same error measure can be used for each loop if it is suitable, but for each parameter, other elements must be duplicated.

Sometimes, a plant has several local minimum in its IP characteristic. In that case, a special measure is taken to find the true minimum. But generally, all plants have a single minimum around the optimum parameter value. The sinusoidal perturbation method can also be applicable to an adaptive system which searches for a maximum IP value.

3. The Analysis of Signals in the Adaptive System

3.1 Block diagram of an adaptive system

To set up a block diagram of a single dimensional parameter perturbation adaptive control system, the circuit models of each component are established. Referring to Fig. 1, the parameter perturbation signal perturbs a plant parameter and the parameter itself also is changing its value due to environment change and parameter correction signal coming through adaptive loop. This parameter perturbation signal and parameter drift cause the error output while the plant and model are operating with input, R. It is necessary to find out an equivalent transfer function block between the parameter perturbation input and the error output. The parameter drift is also an input for the error output and the input is same as perturbation input. We may designate this equivalent transfer function as error measurement transfer function and express as

$$G_m(s) = \frac{K_m}{1 + \tau_m s} \qquad (3\text{-}1)$$

K_m and τ_m are decided by the signal transfer characteristic between perturbation input and error output. To find out the transfer characteristic, phase lag and attenuation introduced to this error output with respect to the original perturbation and parameter drift are observed. Although the perturbation and drift do not come into the plant through ordinary input point but through parameter itself, these signals encounter the same phase shift as a signal of the same frequency coming into the plant through the input of the plant, encounteres. This will be confirmed by actual simulation study in Section 4.1. The attenuation factor of this error signal is not the same as the attenuation factor of the signal of same frequency coming through plant input. The attenuation factor to the error

signal depends on the frequency and magnitude of the plant input, R and the characteristic of the plant. It is also governed by its original signal frequency. If we fix the amplitude and frequency of the input to the plant, the attenuation factor is a function of its original perturbation or drift signal frequency assuming the plant characteristic are constant.

From the above discussion, the τ_m is chosen such that the equivalent transfer function gives the same phase lag as the plant introduces to the same signal coming through the plant input.

Next, the K_m is decided by actual measurement of attenuation. In measuring the attenuation, the input to the plant is kept constant and perturb or drift the parameter by some signal and measure the error output, which gives the attenuation factor between parameter drift input and the error output. Let the measured attenuation be A_m. Then

$$A_m = \frac{K_m}{(1 + \tau_m^2 \omega_m^2)} \tag{3-2}$$

where ω_m is a parameter drift frequency used for measuring A_m.

From equation (3-2) K_m can be calculated.

The IP curve is represented by $IP(X) = a_1(X-X_0)^2 + a_2$ as explained in Section 2. A multiplier M_1 is used as squarer and a_1 is the coefficient of the squarer. After the squarer, a_2 is added. A proper choice of a_1 and a_2 gives an approximate representation of any single dimensional IP curve.

The bandpass filter has the following form,

$$G_1(s) = \frac{\omega_1^2 s}{s^2 + 2\zeta\omega_1 s + \omega_1^2} \tag{3-3}$$

The detector M_2 is a multiplier.

The smoothing filter $G_2(s)$ takes various forms depending on the requirement of compensation of the adaptive loop but generally it is a phase lag filter.

The reference signal filter is an arbitrary filter chosen such that phase shift at frequency ω_1 due to this filter is equal to the phase shift introduced to the perturbation signal between the perturbation input and detector through the adaptive loop.

The integrator is usually a D. C. motor and its transfer function is designated as $G_4(s)$.

The complete block diagram of a single dimensional parameter perturbation adaptive control system becomes as Fig. 4.

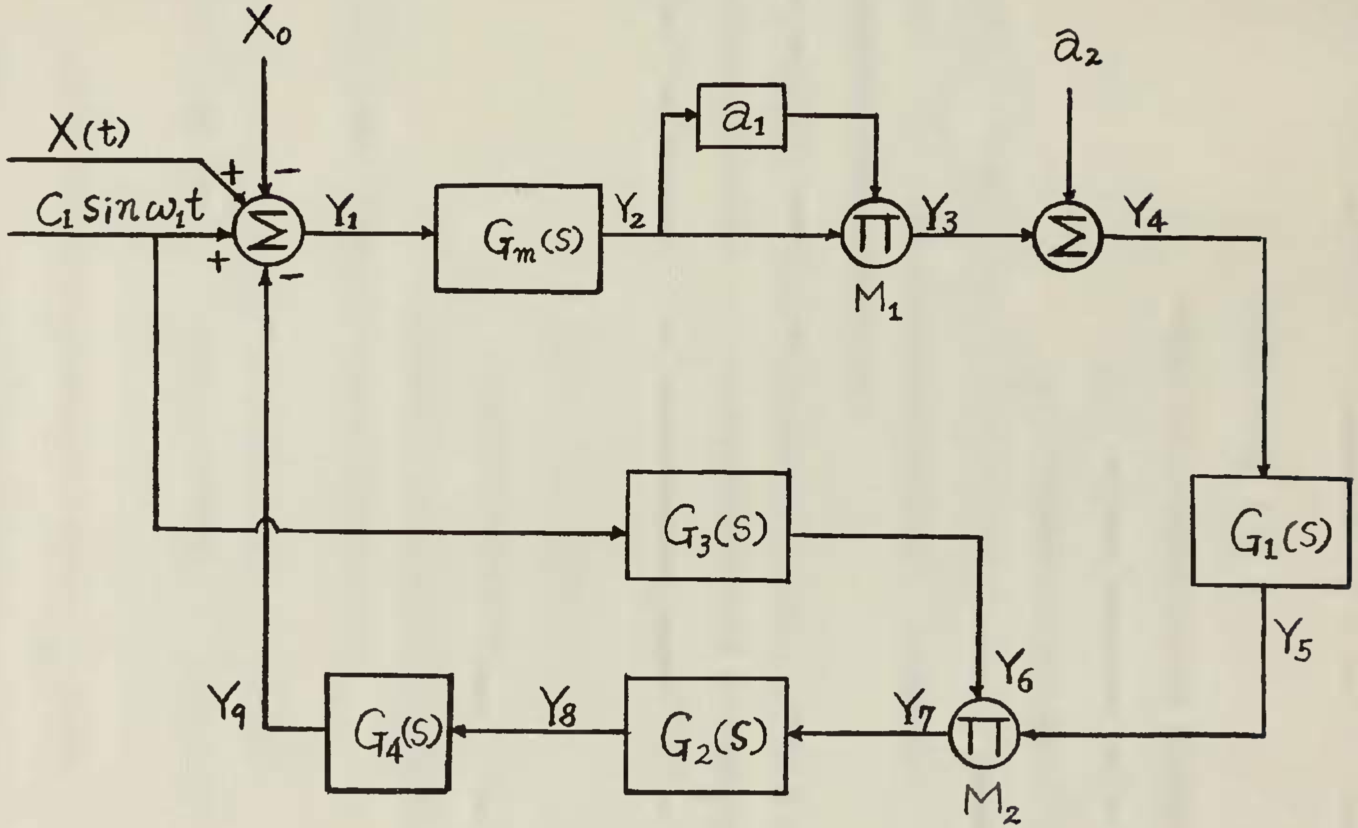


Fig. 4 Block diagram of a single dimensional sinusoidal parameter perturbation adaptive control system.

3.2 Modulation

The input signals through the parameter are the perturbation signal and parameter drift signal due to the variation of the parameter value. The instant error deviation of parameter, $Y_1(t)$ can be expressed as

$$Y_1(t) = X(t) + C_p - X_0 \tag{3-4}$$

where $X(t)$ = instantaneous parameter drift

X_0 = optimum parameter value

C_p = parameter perturbation signal

The analysis of signals will be made for two cases of parameter change; sinusoidal drift of parameter and step drift of parameter.

(a) Sinusoidal drift of parameter

The parameter drifts sinusoidally around the optimum parameter value with very low frequency. $C_p = C_1 \sin \omega_1 t$ is used as perturbation signal throughout the analysis. The instantaneous parameter drift is

$$X(t) = X_0 + X_1 \sin \omega_m t \tag{3-5}$$

ω_m = parameter drift frequency and is much lower frequency than perturbation frequency, ω_1

Then $Y_1(t)$ becomes as below using equation (3-4),

$$Y_1(t) = C_1 \sin \omega_1 t + X_1 \sin \omega_m t \tag{3-6}$$

Referring to the Fig. 4, $Y_1(t)$ goes through $G_m(s)$ and comes out as $Y_2(t)$,

$$Y_2(t) = e(t) = A_p C_1 \sin(\omega_1 t - \theta_p) + A_m X_1 \sin(\omega_m t - \theta_m) \tag{3-7}$$

where A_p, A_m are associated attenuation factors, and θ_p, θ_m are associated phase shifts.

Signals at each point in the loop are labeled in the Fig. 4.

Following the loop,

$$Y_3(t) = a_1 \left[Y_2(t)\right]^2 \tag{3-8}$$

$$Y_4(t) = a_2 + Y_3(t) \tag{3-9}$$

Calculating above equations, $Y_4(t)$ becomes

$$\begin{aligned} Y_4(t) = a_2 + a_1\Big(\frac{A_p^2 C_1^2}{2} + \frac{A_m^2 X_1^2}{2}\Big) - \frac{a_1 A_p^2 C_1^2}{2}\cos 2(\omega_1 t - \theta_p) \\ - \frac{a_1 A_m^2 X_1^2}{2}\cos 2(\omega_m t - \theta_m) \qquad (3\text{-}10) \\ + 2a_1 A_m A_p C_1 X_1 \sin(\omega_m t - \theta_m)\sin(\omega_1 t - \theta_p) \end{aligned}$$

The last term of $Y_4(t)$, $2a_1 A_m A_p C_1 X_1 \sin(\omega_m t - \theta_m)\sin(\omega_1 t - \theta_p)$ is the modulated signal term where the perturbation signal is amplitude modulated by parameter drift signal. This term carries not only the information of magnitude of drift signal but also the information about the actual operating point of parameter, in other words, the actual parameter is higher or lower than the optimum parameter value by the following form in an actual signal.

When $X(t) - X_0 > 0$ or $X_1 \sin(\omega_m t - \theta_m) > 0$, this term is

$$2a_1 A_m A_p C_1 \left| X_1 \sin(\omega_m t - \theta_m) \right| \sin(\omega_1 t - \theta_p) \tag{3-11}$$

When $X(t) - X_0 < 0$ or $X_1 \sin(\omega_m t - \theta_m) < 0$, this term is

$$2a_1 A_m A_p C_1 \left| X_1 \sin(\omega_m t - \theta_m) \right| \sin(\omega_1 t - \theta_p - \pi) \tag{3-12}$$

This is explained in Section 2 and shown in Fig.3.

(b) Step drift of parameter

The parameter value drifts in step from optimum parameter value to some value X, then by equation (3-4), the instantaneous deviation of parameter becomes

$$Y_1(t) = X + C_p - X_0 = (X - X_0) + C_1 \sin\omega_1 t \qquad (3\text{-}13)$$

Going through the equivalent transfer block, $G_m(s)$ and comes out as

$$Y_2(t) = e(t) = A_d(X - X_0) + A_p C_1 \sin(\omega_1 t - \theta_p) \qquad (3\text{-}14)$$

where A_d, A_p are associated attenuation factors and

θ_p is the phase shift.

Calculating the same way as in case (a) using equations (3-8) and (3-9), $Y_4(t)$ comes out as

$$Y_4(t) = a_2 + a_1 \left[A_d^2 (X - X_0)^2 + \frac{A_p^2 C_1^2}{2} \right] - \frac{a_1 A_p^2 C_1^2}{2} \cos\omega_1 t$$

$$+ 2a_1 A_d A_p C_1 (X - X_0) \sin(\omega_1 t - \theta_p) \qquad (3\text{-}15)$$

In $Y_4(t)$ the last term, $2a_1 A_d A_p C_1 (X - X_0)\sin(\omega_1 t - \theta_p)$ is an amplitude modulated signal term where the carrier is the perturbation signal, $\sin(\omega_1 t - \theta_p)$. This term carries the information about the amount of parameter drift $(X-X_0)$ and furthermore the direction of parameter drift from the optimum parameter value. This term takes following form for actual signals:

When $X - X_0 > 0$ or the actual parameter is higher than the optimum parameter value, this term becomes

$$2a_1 A_d A_p C_1 \left| X - X_0 \right| \sin(\omega_1 t - \theta_p) \qquad (3\text{-}16)$$

When $X - X_0 < 0$ or the actual parameter is lower than X_0, this becomes

$$2a_1A_dA_pC_1 \left| X - X_0 \right| \sin(\omega_1 t - \theta_p - \pi) \tag{3-17}$$

as explained in Section 2 and shown in Fig. 3.

3.3 Filtering

The bandpass filter $G_1(s)$ attenuates considerably all unnecessary signals in $Y_4(t)$ and only passes the components that contain information about the parameter drift, without much attenuation.

(a) Taking the case of sinusoidal parameter drift, the $Y_4(t)$, equation (3-10) in Section 3.2(a), is rewritten as

$$Y_4(t) = D_0 - D_1\cos 2(\omega_1 t - \theta_p) = D_2\cos 2(\omega_m t - \theta_m)$$

$$+ D_3\sin(\omega_m t - \theta_m)\sin(\omega_1 t - \theta_p) \qquad (3\text{-}18)$$

where

$$D_0 = a_2 + \frac{a_1}{2}(A_p^2C_1^2 + A_m^2X_1^2)$$

$$D_1 = \frac{a_1A_p^2C_1^2}{2}$$

$$D_2 = \frac{a_1A_m^2X_1^2}{2}$$

$$D_3 = 2a_1A_mA_pC_1X_1$$

Expanding the last term of equation (3-18),

$$Y_4(t) = D_0 - D_1\cos 2(\omega_1 t - \theta_p) - D_2\cos 2(\omega_m t - \theta_m)$$

$$+ \frac{D_3}{2}\cos\left[(\omega_1 - \omega_m)t - \theta_p + \theta_m\right] - \frac{D_3}{2}\cos\left[(\omega_1 + \omega_m)t - \theta_p - \theta_m\right] \qquad (3\text{-}19)$$

The $Y_4(t)$ contains the following frequency signal components,

D.C., $2\omega_m$, $2\omega_1$, $\omega_1 - \omega_m$, $\omega_1 + \omega_m$.

The necessary intelligence is contained in $\omega_1 - \omega_m$ and $\omega_1 + \omega_m$ components. Therefore the bandpass filter is designed to pass these two components without attenuation and suppress other components. Thus the center frequency of the bandpass filter should be ω_1 and this filter does not introduce any phase shift to the signal of ω_1. Assuming that the phase angle characteristic is linear near the center frequency, which is generally true for an actual filter, the signal of $(\omega_1 - \omega_m)$ gets $+\Delta\theta$ phase shift and the signal of $(\omega_1 + \omega_m)$ gets $-\Delta\theta$ phase shift with respect to the signal of ω_1. Then $Y_4(t)$ passing the filter, becomes as

$$Y_5(t) = A_0D_0 - A_1D_1\cos 2(\omega_1 t - \theta_p - \theta_1) - A_2D_2\cos 2(\omega_m t - \theta_m - \theta_2)$$

$$+ \frac{A_3D_3}{2}\cos\left[(\omega_1 - \omega_m)t - \theta_p + \theta_m + \Delta\theta\right]$$

$$- \frac{A_4D_3}{2}\cos\left[(\omega_1 + \omega_m)t - \theta_p - \theta_m - \Delta\theta\right] \qquad (3\text{-}20)$$

where A_0, A_1, A_2, A_3, and A_4 are attenuation factors associated eith each signal and

θ_1, θ_2 and $\Delta\theta$ are additional phase shifts introduced to each signal. Usually in a band pass filter A_3 and A_4 are the same attenuation as the center frequency signal suffers because $\omega_m \ll \omega_1$ as assumed in Section 3.2. A_0, A_1 and A_2 should be much smaller than A_3 for our purpose. Combining the last two terms of equation (3-20), $Y_5(t)$ becomes

$$Y_5(t) = A_0D_0 - A_1D_1\cos 2(\omega_1 t - \theta_p - \theta_1) - A_2D_2\cos 2(\omega_m t - \theta_m - \theta_2)$$

$$+ A_3D_3\sin(\omega_1 t - \theta_p)\sin(\omega_m t - \theta_m - \Delta\theta) \qquad (3\text{-}21)$$

If the modulated signal term in $Y_4(t)$, equation (3-18) is compared with the modulated signal in $Y_5(t)$, equation (3-21), it is noted that the band-pass filter introduced a phase lag of $\Delta\theta$ only to the modulating signal component, $(\sin \omega_m t - \theta_m)$ as far as the modulating signal term is concerned.

(b) In step parameter drift case, $Y_4(t)$, equation (3-15) is rewritten as

$$Y_4(t) = D_5 - D_6 \cos 2\omega_1 t + D_7 (X - X_0) \sin(\omega_1 t - \theta_p) \tag{3-22}$$

where $D_5 = a_2 + a_1 \left[A_d^2 (X - X_0)^2 + \frac{A_p^2 C_1^2}{2} \right]$

$$D_6 = \frac{a_1 A_p^2 C_1^2}{2}$$

$$D_7 = 2a_1 A_d A_p C_1$$

Passing the bandpass filter, $Y_4(t)$ becomes

$$Y_5(t) = A_0 D_5 - A_1 D_6 \cos(2\omega_1 t - \theta_2) + A_3 D_7 (X - X_0) \sin(\omega_1 t - \theta_p) \tag{3-23}$$

3.4 Demodulation

The modulated signal $Y_5(t)$ reaches the detector M_2, where $Y_5(t)$ is multiplied by the detection reference signal and demodulation is accomplished. The detection reference signal should be in the same phase as the carrier signal to get maximum output at the demodulation. For this purpose, detection reference signal phase lag filter $G_3(s)$ is used to give the reference signal an exactly same phase lag as the phase lag introduced to the carrier signal between the perturbation input at the plant and the detector; in our case which is θ_p. Thus the detection reference signal has the following form,

$$Y_6(t) = BC_1 \sin(\omega_1 t - \theta_p) \tag{3-24}$$

where B = attenuation factor of $G_3(s)$

θ_p = phase lag introduced by $G_3(t)$

(a) For the sinusoidal parameter drift case, $Y_5(t)$ equation (3-21) is multiplied by $Y_6(t)$ equation (3-24) and we get

$$\begin{aligned} Y_7(t) = {} & A_0 D_0 B C_1 \sin(\omega_1 t - \theta_p) - A_1 D_1 B C_1 \cos 2(\omega_1 t - \theta_p - \theta_1) \\ & \sin(\omega_1 t - \theta_p) - A_2 D_2 B C_1 \cos 2(\omega_m t - \theta_m - \theta_2) \sin(\omega_1 t - \theta_p) \\ & + A_3 D_3 B C_1 \sin^2(\omega_1 t - \theta_p) \sin(\omega_m t - \theta_m - \Delta\theta) \end{aligned} \tag{3-25}$$

In $Y_7(t)$ only a D.C. component is integrated by the integrator following and develops a parameter correction signal. Only the last term of $Y_7(t)$ contains a D. C. component which is varying slowing in accordance with the modulating signal, which is the parameter drift. All other terms do not contribute

to the parameter correction because they are comparatively high frequency sinusoids and furthermore, A_0, A_1 and A_2 are very small due to the band-pass characteristic of $G_1(s)$ and they will be further attenuated by the smoothing filter following. If only the last term of $Y_7(t)$ is expanded it becomes

$$\frac{A_3D_3B\ C_1}{2}\sin(\omega_m t - \theta_m - \Delta\theta) - \frac{A_3D_3B\ C_1}{2}\sin(\omega_m t - \theta_m - \Delta\theta)$$

$$\cos 2(\omega_1 t - \theta_p) \tag{3-26}$$

The first term of (3-26) is considered as a slowly varying D. C. which corrects the parameter drift. Thus,

$$Y_{7d.c.}(t) = \frac{A_3D_3B\ C_1}{2}\sin(\omega_m t - \theta_m - \Delta\theta) \tag{3-27}$$

Putting back the original value for D_3 and taking into account the phase shift between $Y_7(t)$ and $Y_9(t)$,

$$Y_9(t) = a_1A_mA_pA_4B\ C_1^2X_1\sin(\omega_m t - \theta_m - \Delta\theta - \theta_3) \tag{3-28}$$

where θ_3 is the phase shift in $G_2(s)$ and $G_4(s)$.

A_4 is the attenuation factor associated

The parameter correction signal $Y_9(t)$ has the same form as the parameter drift and has some additional phase lag compared to the parameter drift. This correction signal will be fed back negatively to parameter adjustment and corrects the drifted parameter to the optimum parameter value.

(b) For the step parameter drift case, $Y_5(t)$ equation (3-23) is multiplied by $Y_6(t)$ equation (3-24) and we get

$$Y_7(t) = A_0D_5B\ C_1\sin(\omega_1 t - \theta_p) - A_1D_6B\ C_1\cos(2\omega_1 t - \theta_2)$$

$$+ A_3 D_7 B\ C_1 (X - X_0) \sin^2(\omega_1 t - \theta_p) \tag{3-29}$$

In $Y_7(t)$ only the last term develops a D. C. component, which will be integrated and drive the parameter adjustment. Other terms do not contribute to parameter correction because they are comparatively high frequency sinusoids and A_0 and A_1 are very small due to the bandpass filter attenuation and will be further attenuated by the smoothing filter. If only the last term of $Y_7(t)$ equation (3-29) is expanded it becomes as,

$$\frac{1}{2} A_3 D_7 B\ C_1 (X-X_0) - \frac{1}{2} A_3 D_7 B\ C_1 (X-X_0) \cos 2(\omega_1 t - \theta_p) \tag{3-30}$$

Putting back the original value for D_7,

$$Y_{7d.c.}(t) = a_1 A_d A_p A_3 B C_1^2 (X - X_0) \tag{3-31}$$

Thus the rate at which the parameter drift is corrected, is proportional to the parameter error $(X-X_0)$, the perturbation signal amplitude C_1, the constant a_1 associated with the IP curve (the sensitivity of the IP variation to changes in parameter value) and adaptive loop gain. This open loop analysis is not exactly correct when the loop is closed, but it shows a possible signal development through the adaptive loop.

(c) Next, the effect of phase difference between the carrier and the detection reference signal $Y_6(t)$, in the demodulation, is considered. To see more clearly the process of demodulation and the effect of the phase difference between carrier and the detection reference signal, graphical multiplication of actual signals involved, is performed. Taking the case of step parameter drift, the actual modulated signal in $Y_5(t)$ has the following forms as explained in 3.2(b) and expressed in equations (3-16) and (3-17),

$$A \left| X - X_0 \right| \sin(\omega_1 t - \theta_p) \quad \text{when} \quad X - X_0 > 0 \tag{3-32}$$

$$A \left| X - X_0 \right| \sin(\omega_1 t - \theta_p - \pi) \quad \text{when} \quad X - X_0 < 0 \tag{3-33}$$

where A is a constant.

The carrier in $Y_5(t)$ is $C_{y5}(t) = \sin(\omega_1 t - \theta_p)$ when $X-X_0 > 0$ and $-C_{y5}(t) = \sin(\omega_1 t - \theta_p - \pi)$ when $X-X_0 < 0$.

The detection reference signal is $Y_6(t) = B\, C_1 \sin(\omega_1 t - \theta_r)$.

If $\theta_r \neq \theta_p$ $Y_6(t)$ leads or lags the carrier $C_{y5}(t)$ by some angle. The graphical multiplication is performed for the following three cases,

(1) $Y_6(t)$ is in phase with the carrier.

(2) $Y_6(t)$ leads the carrier by 30°.

(3) $Y_6(t)$ lags the carrier by 30°.

The coefficients of each term is a constant, therefore only a sine wave of unit maximum amplitude represents each wave.

Fig. 5 shows the demodulated output by dotted line for the above three cases. It shows clearly the phase sensitive nature of perturbation signal; when the carrier in a modulated signal is $C_{y5}(t) = \sin(\omega_1 t - \theta_p)$, the demodulated output is all positive and when the carrier is $-C_{y5}(t) = \sin(\omega_1 t - \theta_p - \pi)$ the demodulated output is all negative. Thus it will give correct direction of parameter correction in accordance with the actual parameter error, $X-X_0 > 0$ or $X-X_0 < 0$.

In case (1), we get maximum net output of correction signal when integrated as seen in Fig. 5(1). For the case (2), we get less net output of correction signal when integrated. This eventually ends up with less adaptive loop gain. The case (3) has the same output as in the case (2).

If we consider the time lag of starting to build up correction signal

output at the integrator, the case (3) has the largest time lag of the three cases, and the case (1) is the second and the case (2) is the least. But this time lag is negligible because the carrier frequency is very high compared to the modulating frequency.

From this analysis, we can see that when the phase difference between Y_5 and Y_6 is exactly 90°, the signal output will be the lowest and when the phase difference is more than 90° the demodulated signal polarity will be reversed and the magnitude will increase as the phase difference approaches 180°. Therefore, when the carrier phase lag is expected to be closer to 180°, it is better to remove the reference signal phase lag filter and change the polarity of the demodulated output to have correct feedback.

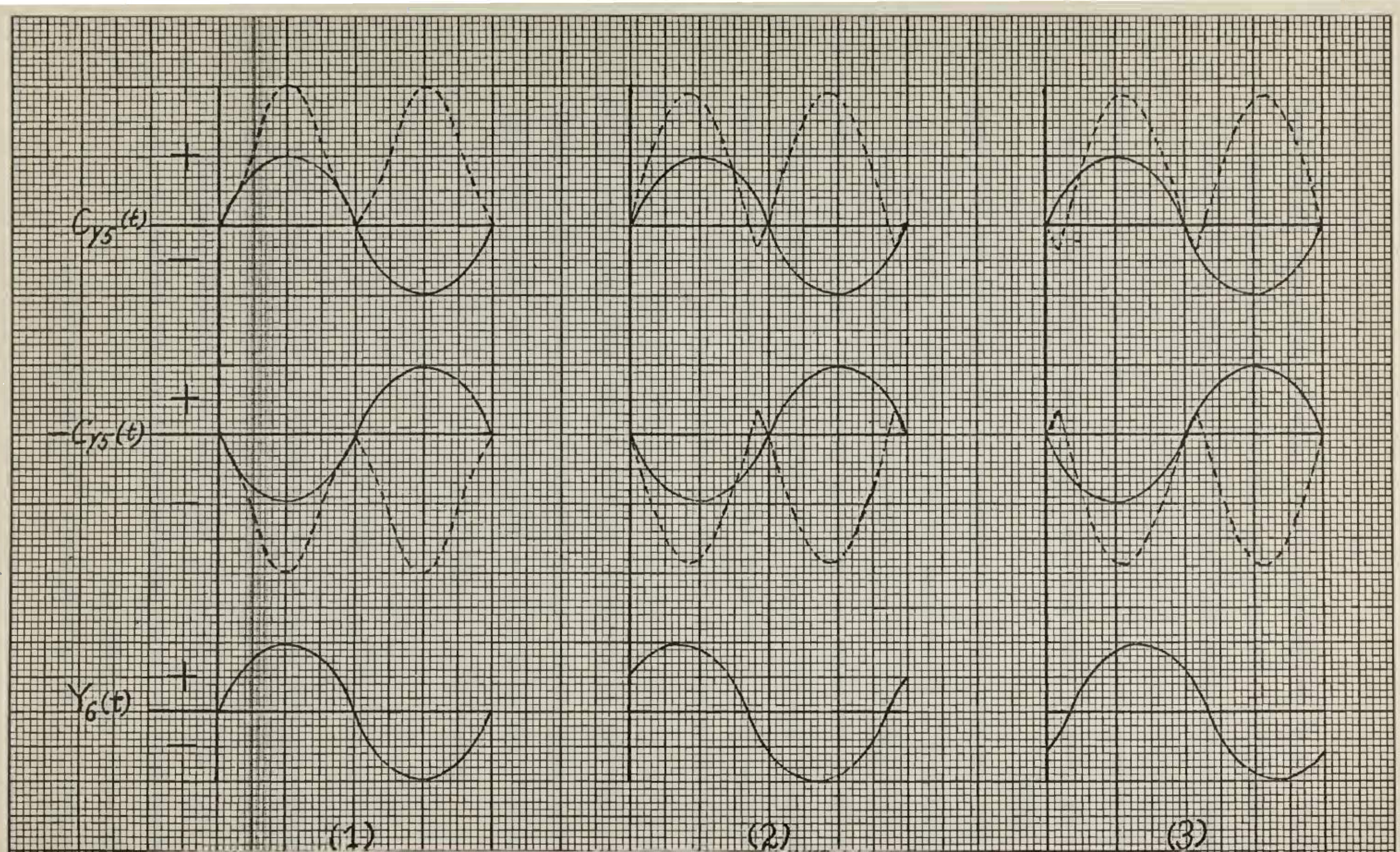


Fig. 5 Process of demodulation (solid line is carrier and dotted line is demodulated output)

(1) $Y_6(t)$ is in phase with $C_{y5}(t)$. (2) $Y_6(t)$ leads $C_{y5}(t)$ by 30°.

(3) $Y_6(t)$ lags $C_{y5}(t)$ by 30°.

4. The Simulation Study of a Sinusoidal Perturbation System

4.1 IP. variation investigation

To investigate the actual variation of the IP. with respect to the parameter drift and perturbation of the plant, the following model and plant were simulated by an analog computer with the adaptive loop opened. The analog computer used throught the investigation is the Donner 3100.

$$\text{Model;} \quad G_{mod}(s) = \frac{K}{S(S + X_0)}$$

$$\text{Plant;} \quad G_p(s) = \frac{K}{S(S + x)}$$

The pole of the plant is a variable parameter x, and the pole of the model is fixed at 3, which is the optimum parameter value in this case and gives ζ of 0.5. The pole of the plant can vary in the range that gives ζ of plant from 0.1 to 1. with K = 9. The block diagram of the simulation is as Fig. 6 and the analog computer circuit is shown in Fig. 7.

(a) The input to the plant and model is a square wave of 0.1 cps. A plant parameter, the pole of the plant is drifted by a triangular wave of 0.01 cps. while the model and plant are operating. The output of model and plant are as Fig. 8(a) and (b) respectively. The output wave shape shows the change of ζ of the plant due to the parameter drift.

(b) The error output is recorded as Fig. 8(c) and the pole drift is as Fig. 8(d).

Next, the input to plant and model is sine wave of 0.3 cps. and the error output is also recorded as Fig. 9(a). The Fig. 9(b) is the pole drift as before and (c) is the error squared output.

The above results shows that error is produced only when the output of plant is in transient and the plant response is not the same as the model response due to the parameter drift. Also it shows that the magnitude

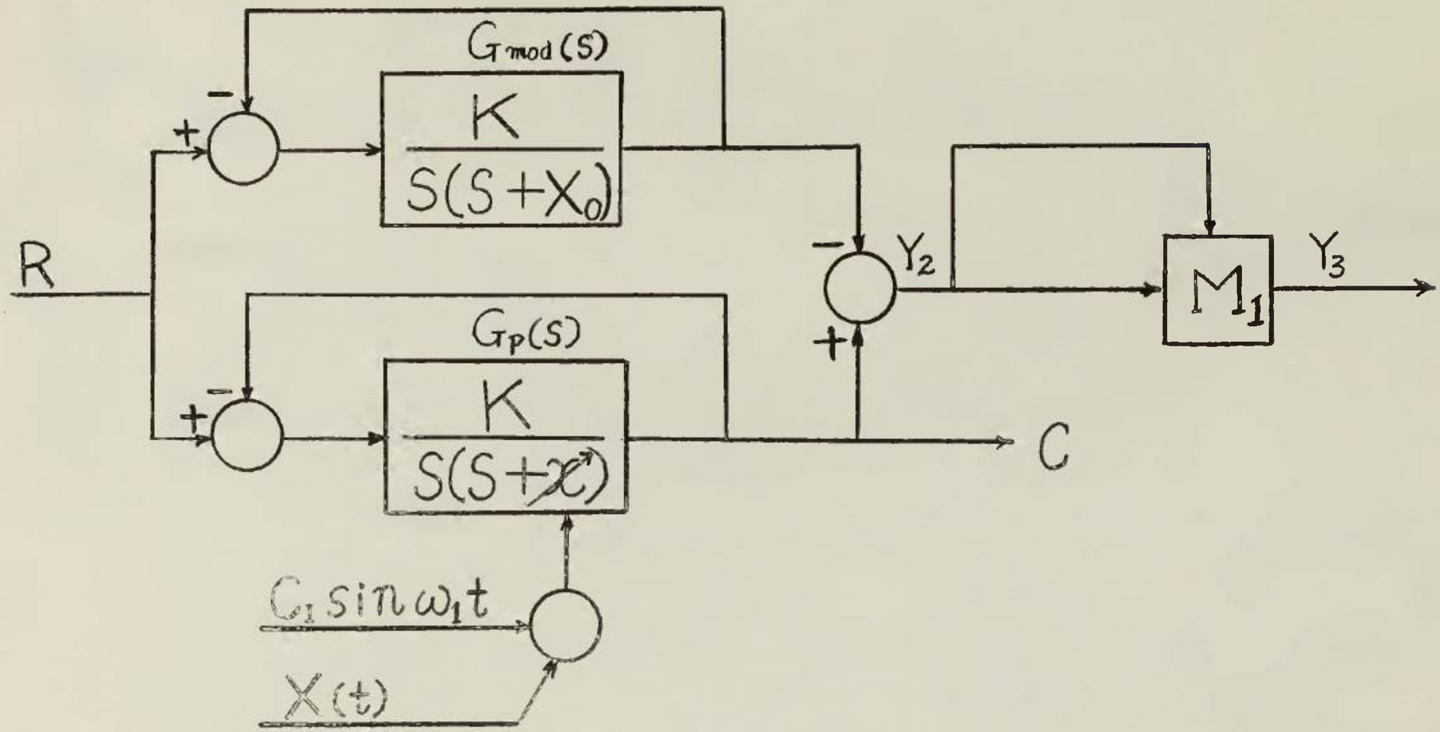


Fig. 6 The model and plant set-up for the investigation of IP.

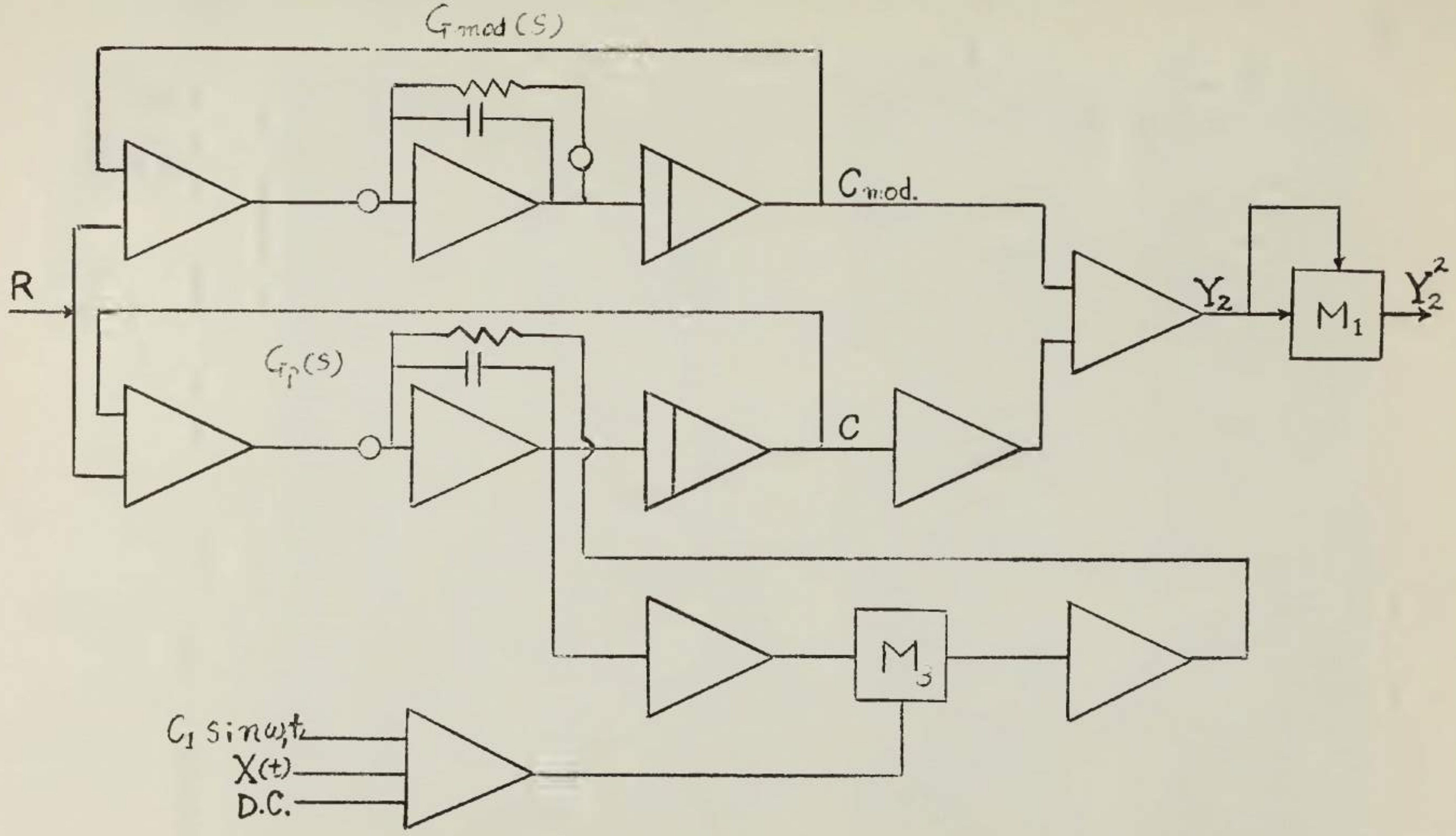


Fig. 7 The analog computer circuit of the model and plant set-up for the investigation of IP.

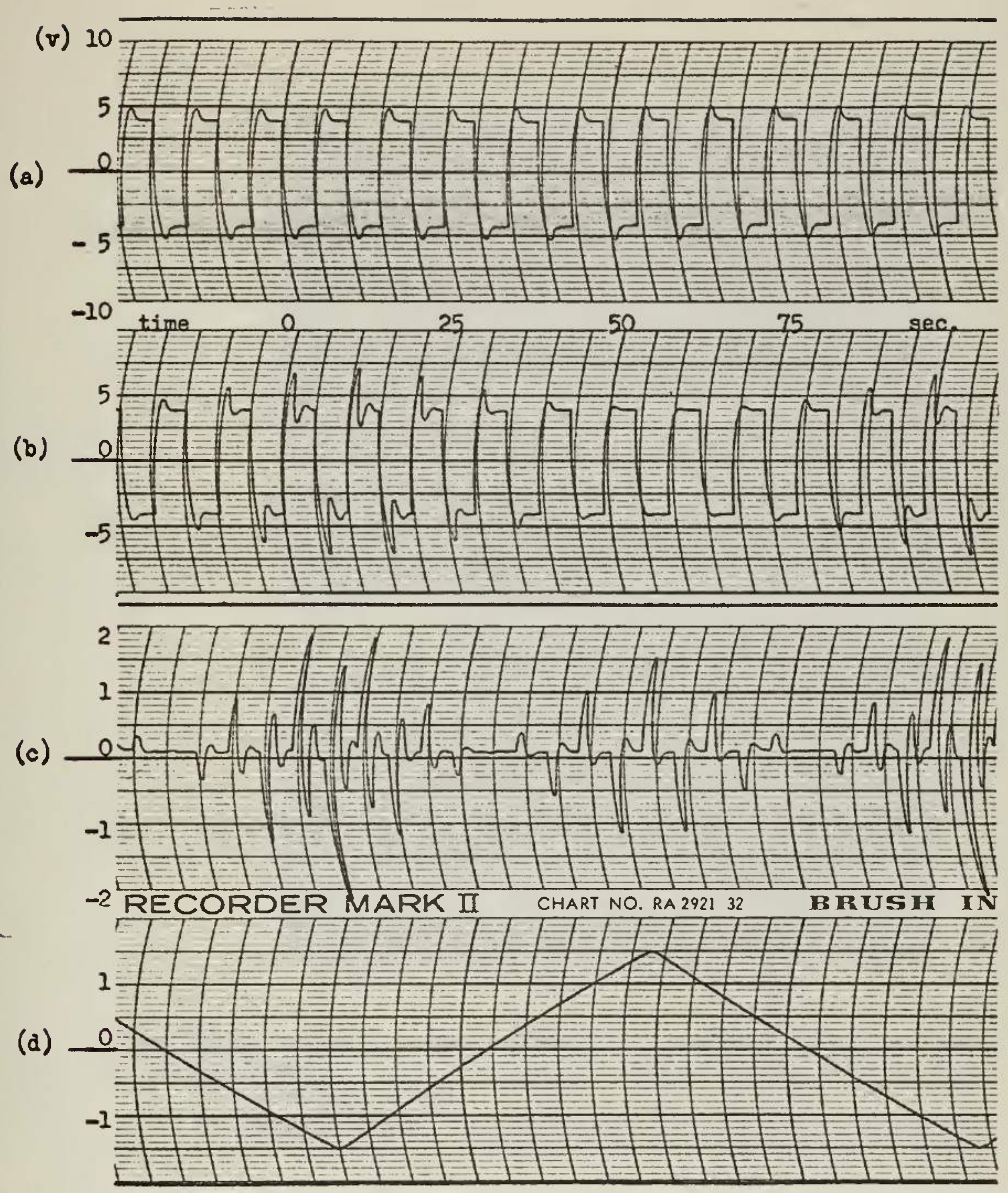


Fig. 8. The output of model, plant and error with plant pole drifted.
(a) model,
(b) plant
(c) error
(d) plant pole drift

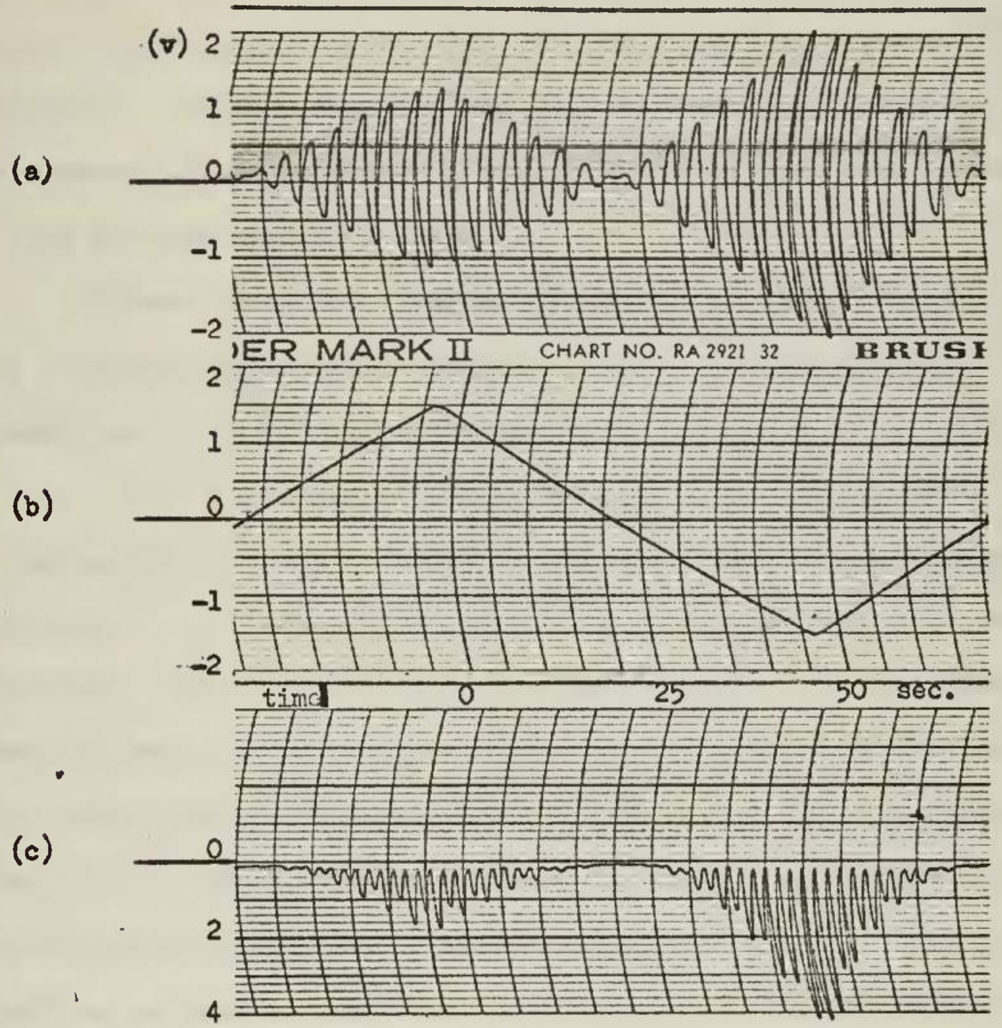


Fig. 9. Error and error squared output with sine wave input to plant.
(a) error
(b) plant pole drift
(c) error squared

of error is proportional to the parameter error, the magnitude of parameter drift.

The magnitude of error is bigger when the parameter drift is in the direction of decreasing plant ζ with the same amount of parameter drift. Therefore, it is expected that the magnitude of parameter correction signal will be greater for the parameter drift to a decreasing ζ than a increasing ζ, with the same amount of drift.

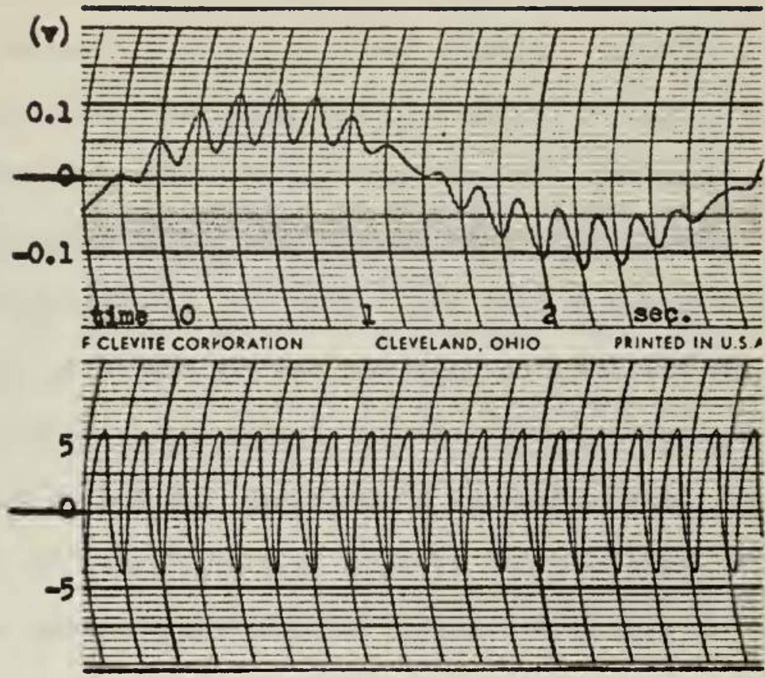
(c) Error output is observed with only perturbing the pole of plant by a sinusoidal perturbation signal. The perturbation frequency is much higher than a drift frequency; here a sine wave of 5 cps is used.

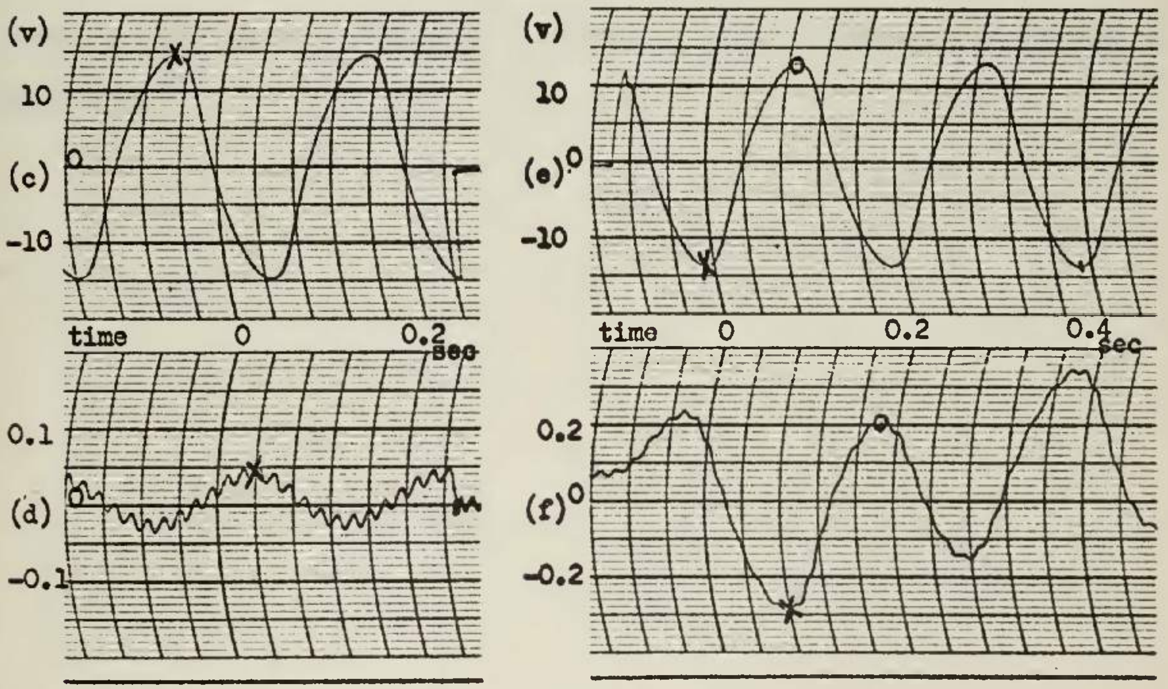
(1) The magnitude of error output due to perturbation signal only. The perturbation wave appears in the error output, but the magnitude is much reduced due to the severe attenuation. Five cps is a high frequency for this plant. Thus the parameter perturbation signal does not appear conspicuously in the plant output even with a big amplitude perturbation signal as far as the frequency of perturbation is high. If we want to eliminate the effect of perturbation on plant output completely, the parameter of model can be perturbed instead of plant parameter. Fig. 10(a) is the perturbation wave appearing on the output of the plant. The perturbation waves appear conspicuously on the output of the plant only when the ratio of the amplitude of the plant input to the amplitude of the perturbation signal is small.

(2) Phase shift of perturbation signal through plant. The perturbation wave appearing on the output of plant lags the original perturbation signal by some angle. This phase lag is introduced by the plant and depends on the characteristic of plant and signal frequency. Fig. 10(c) and (d) are the original perturbation signal and the perturbation

(a) Perturbation wave appearing on plant output.

(b) Original perturbation signal.





(c) Original perturbation.
(d) perturbation wave on output

(e) Input to plant
(f) Output of plant

Fig. 10. Perturbation waves on the output of plant.

wave appearing on the plant output respectively. The same marks on the waves are the corresponding points on both waves. These corresponding points are found by starting and stopping the signal abruptly. According to the measurement of phase difference between the two waves, perturbation wave appearing on the output lags the original signal by 165°. To compare this phase lag with the phase lag introduced by plant to a signal coming into the plant through the plant input, the same signal used as perturbation is fed into the plant. Fig. 10(e) and (f) are the input to plant and the output of plant. The same marks on the signals are the corresponding points on both waves which is found as before. According to the measurement, the output waves lags input by 165° approximately, which is the same phase lag introduced by plant to the parameter perturbation signal. From this result it is concluded that the phase lag introduced by the plant to the parameter perturbation signal fed through the pole of plant is the same phase lag introduced by plant to a signal of same frequency fed through the plant input.

4.2 The Simplest Adaptive System Simulation (Adaptive circuit 1)

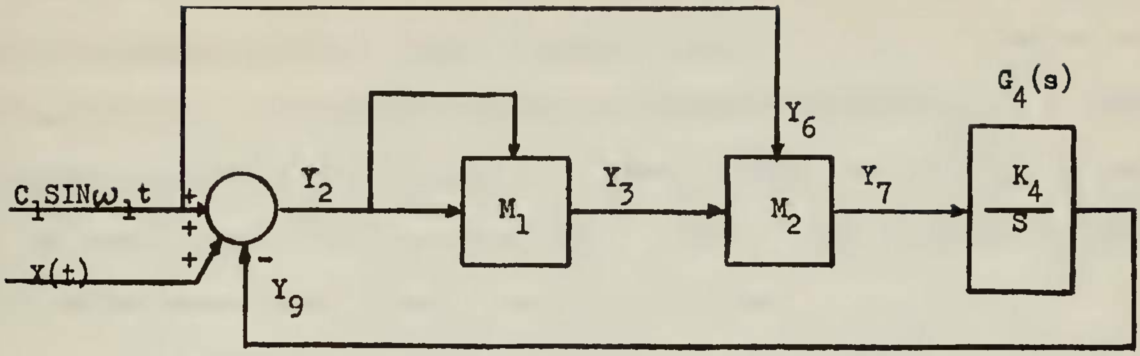
The complete adaptive control system is as Fig. 1 and the block diagram is as Fig. 4. To investigate the function of each component and the effect of change in components on the over-all system operation, the simulation is started with the simplest combination of components for a adaptive system. Then more components are added to improve the system operation until the complete adaptive system is formed. We designate successive circuits by number from the simplest to the complete circuit in simulation study. The first circuit is designated as adaptive circuit 1, which is the simplest and is shown in Fig. 11(a). The analog computer set-up is as Fig. 11 (b). It simulates starting from the error output point in the complete system. In Section 3.2, signal analysis, the instant deviation of parameter $Y_1(t)$ is expressed as equation (3-4),

$$Y_1(t) = X(t) + C_p - X_0 \tag{4-1}$$

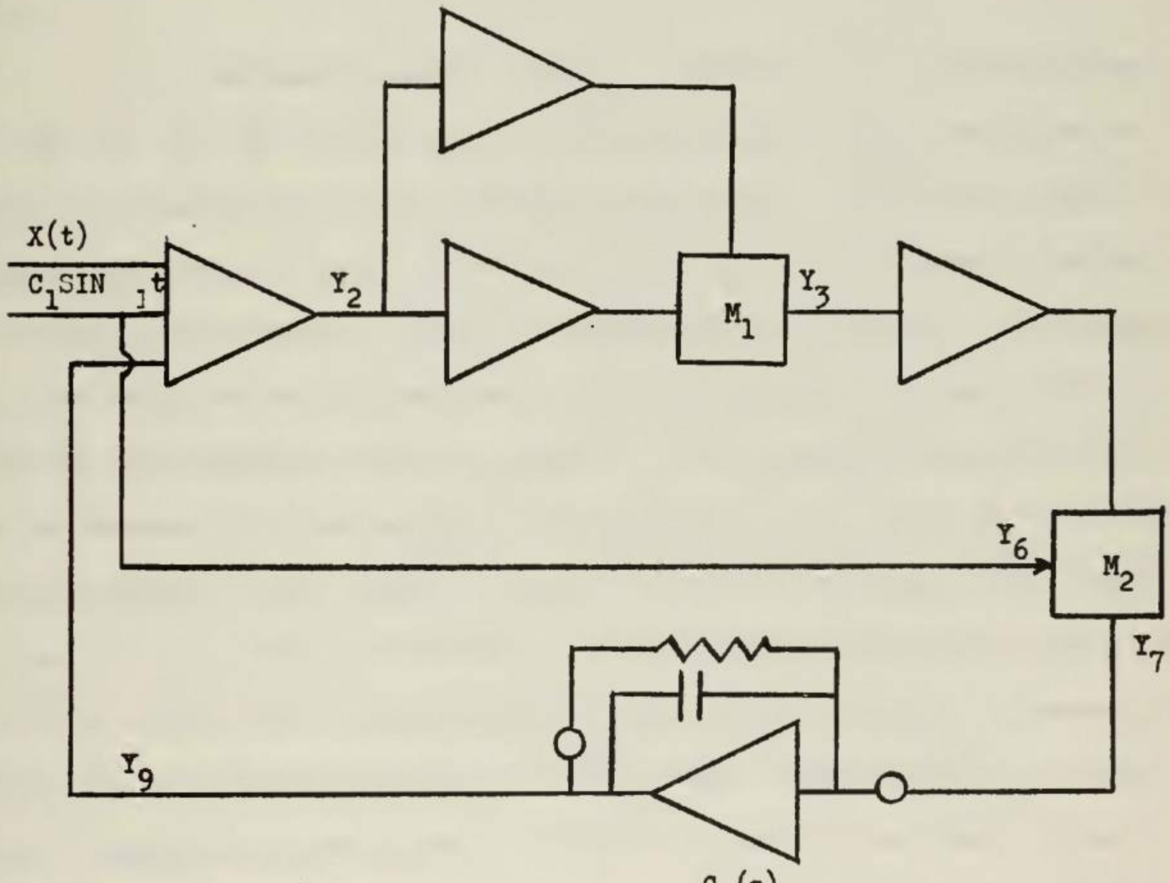
For the analysis the optimum parameter value X_0 is arbitrary and can be set as zero. $Y_1(t)$ causes an error output and the error output $Y_2(t)$ has severe attenuation and phase shift and comes out as equation (3-6) for a sinusoidal parameter drift. But in the simulation study, ignore the attenuation and phase shift and assume the error output $Y_2(t)$ as,

$$Y_2(t) = X(t) + C_p \tag{4-2}$$

This error output is going to be the input in the Adaptive circuit 1 because the circuit simulates from error output point. In an actual system, the parameter correction signal corrects plant parameter and reduces the error output to zero but in this simulation the correction signal directly reduces the $Y_2(t)$, which is equivalent to correcting the plant parameter.



(a) block diagram



(b) computer circuit

Fig. 11. Adaptive circuit 1.

In the circuit, M_1 and M_2 are a squarer and a detector, both of which are electronic multipliers. The zero D. C. level of electronic multiplier is not constant, therefore a pure integrator can not be put in right after the multiplier M_2 because the integrator integrates the drifting D. C. and developes erratic correction signal. A pseudo integrator $\frac{K_4}{S + 0.1}$ is used for the simulation of the pure integrator $\frac{K_4}{S}$. The parameter perturbation and parameter drift signals used are as follows:

$$C_p = C_1 \sin \omega_1 t \qquad C_1 = 1 \text{ v.} \qquad \omega_1 = 5 \text{ cps.}$$

$$X(t) = X_1 \sin \omega_m t \qquad X_1 = 8 \text{ v.} \qquad \omega_m = 0.1 \text{ cps.}$$

The adaptive loop is open at Y_9 feedback point and open loop observation is made.

(a) The wave shapes of signals at every point of loop are shown in Fig. 12. The (a) in the Fig. is the error signal in which perturbation signal is superimposed on the parameter drift signal. (b) is the output of squarer where the modulation is accomplished. (c) is the output of detector; the demodulated output has positive or negative D.C. component in accordance with the parameter drift direction. This is integrated and comes out as (d), which is the parameter correction signal. If the parameter correction signal is compared with the parameter drift signal in (a), it has the same shape as the parameter drift, therefore, when the correction signal is negatively fed back it will correct the parameter drift. But the correction signal lags the drift signal by 60° approximately as seen in (a) and (d). The marked points are the corresponding point on both waves. This phase lag is caused by the integrator as seen between (c) and (d). There is no phase lag between Y_1 and Y_7 as seen in (a), (b) and (c). The perturbation signal appears in the correction signal Y_9, which will be eliminated by adding a

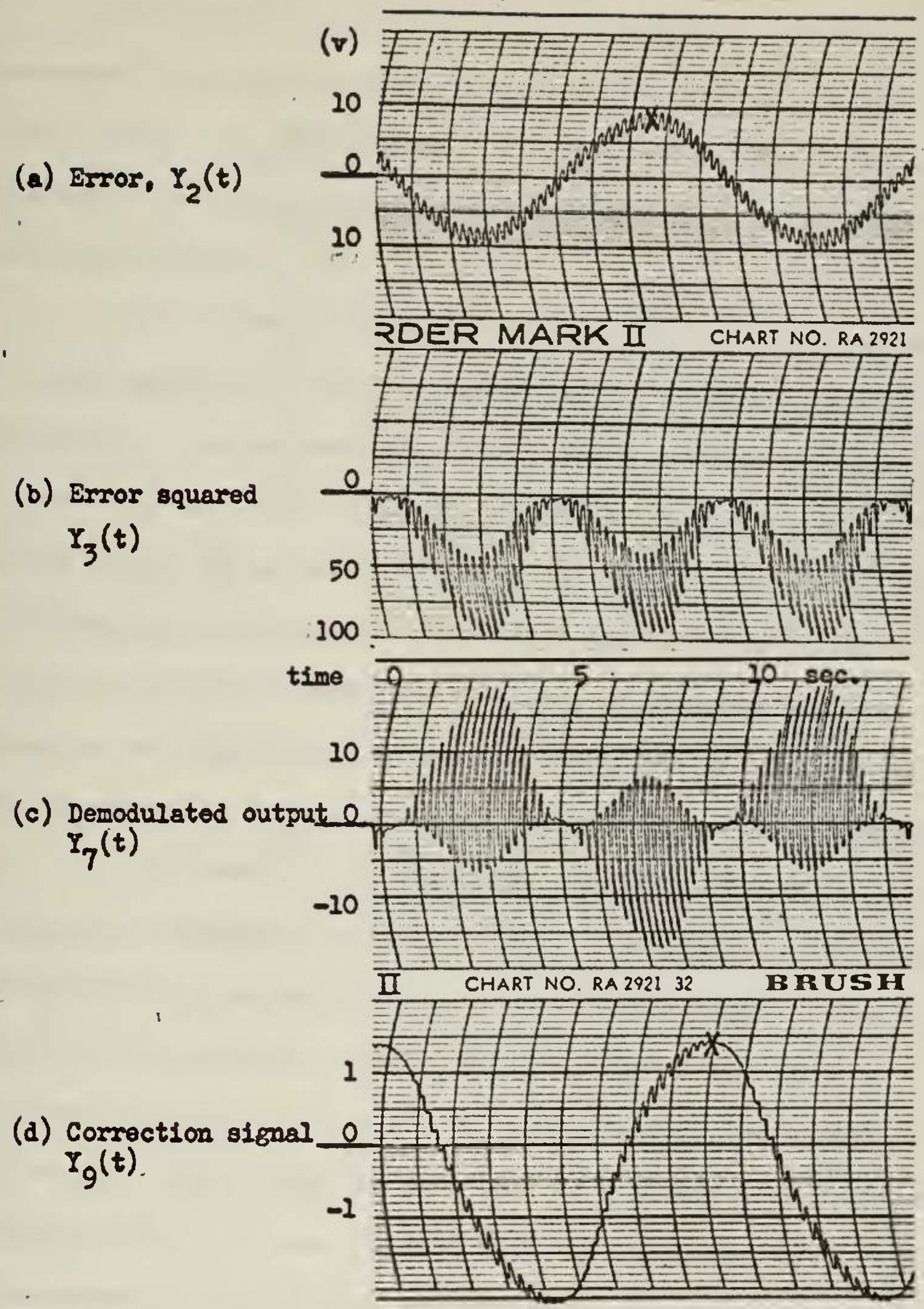


Fig. 12. Wave shapes of signals in Adaptive circuit 1.

smoothing filter.

A square wave also can be used as a parameter perturbation signal. The same results as obtained by sinusoidal perturbation, are obtained using square wave perturbation. The square wave perturbation wave appearing in the correction signal looks like a sinusoidal wave. Fig. 12-1 shows the results.

(b) When the frequency of perturbation is increased, it is attenuated more and it does not appear in the correction signal; in this case there is no perturbation signal in the correction signal when 15 cps. is used for ω_1. The amplitude of the correction signal is not affected by the change of perturbation signal frequency. This means the perturbation frequency does not affect the loop gain in this simple circuit, however, in an actual circuit the frequency of perturbation will affect the amount of attenuation due to circuit components and affect the loop gain. Fig. 13 shows the correction signal with different perturbation frequencies.

(c) Next, the perturbation frequency is fixed at 5 cps. and the frequency of parameter drift is varied. When the drift frequency increases, the amplitude of correction signal is decreased. Fig. 14 is the amplitude of correction signal vs. drift frequency graph. The decrease of amplitude of correction signal is due to the attenuation by the integrator. This result and the result from (b) show that in a closed loop analysis of the adaptive circuit, we need to deal with the parameter drift signal only. The perturbation signal does not affect the magnitude of correction signal even though it comes through the detector and rides on the correction signal.

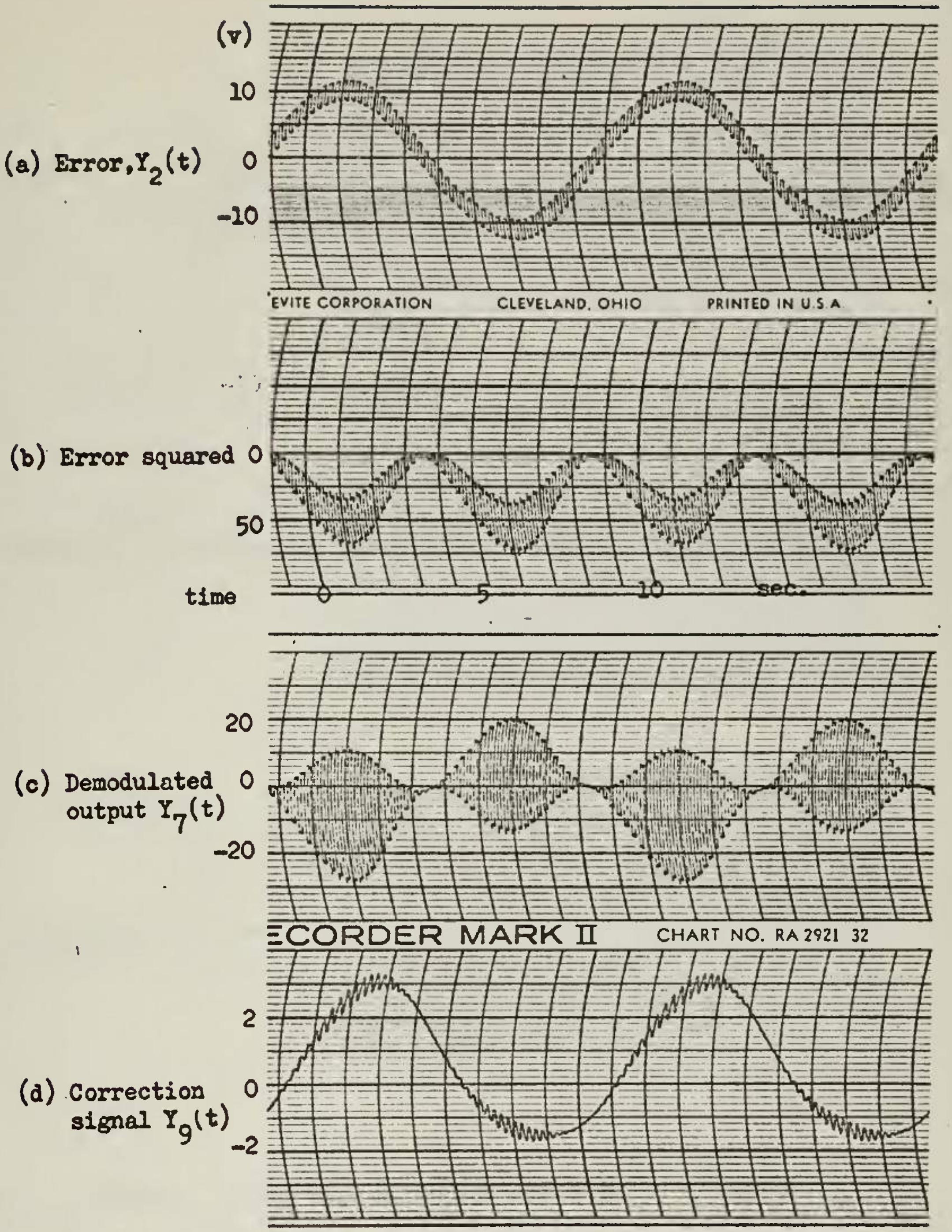
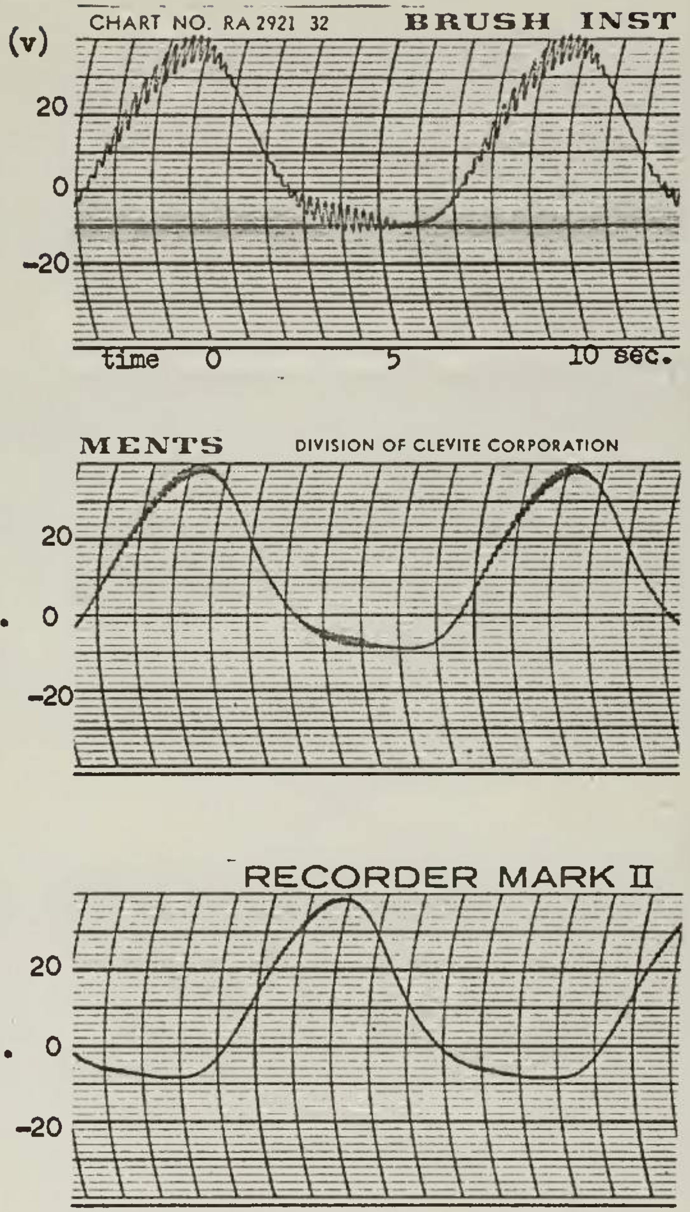


Fig. 12-1 Wave shapes of signals in adaptive circuit 1 with square wave perturbation.



(a) when $\omega_1 = 5$ cps.

(b) when $\omega_1 = 10$ cps.

(c) when $\omega_1 = 15$ cps.

Fig. 13. Correction signal with different perturbation frequency.

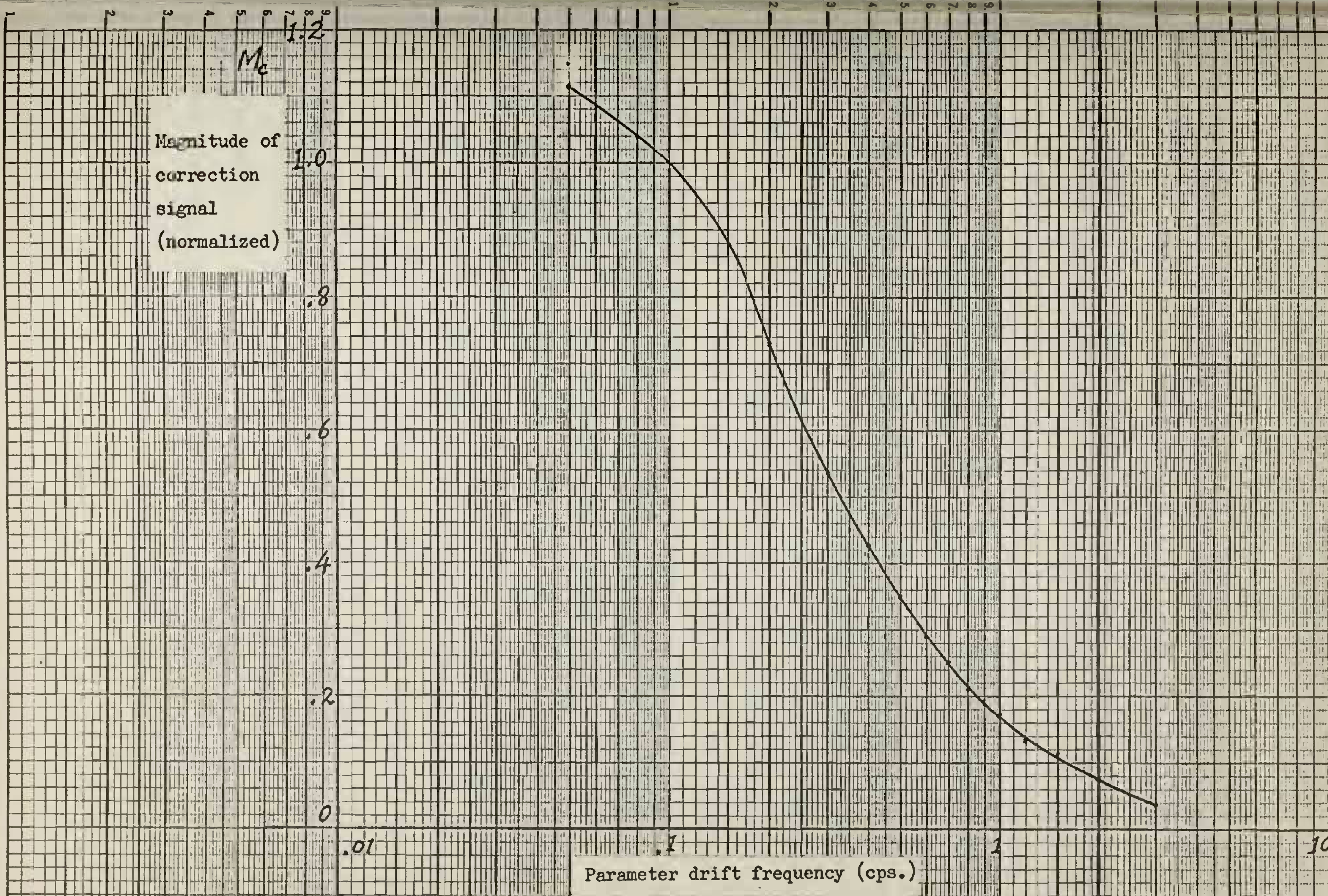
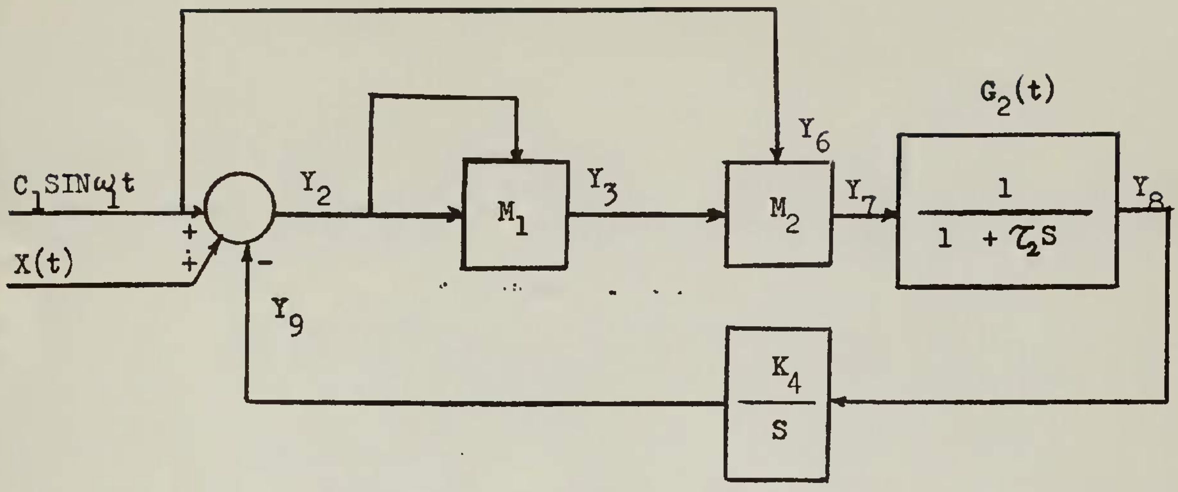


Fig. 14 The magnitude of correction signal vs. parameter drift frequency

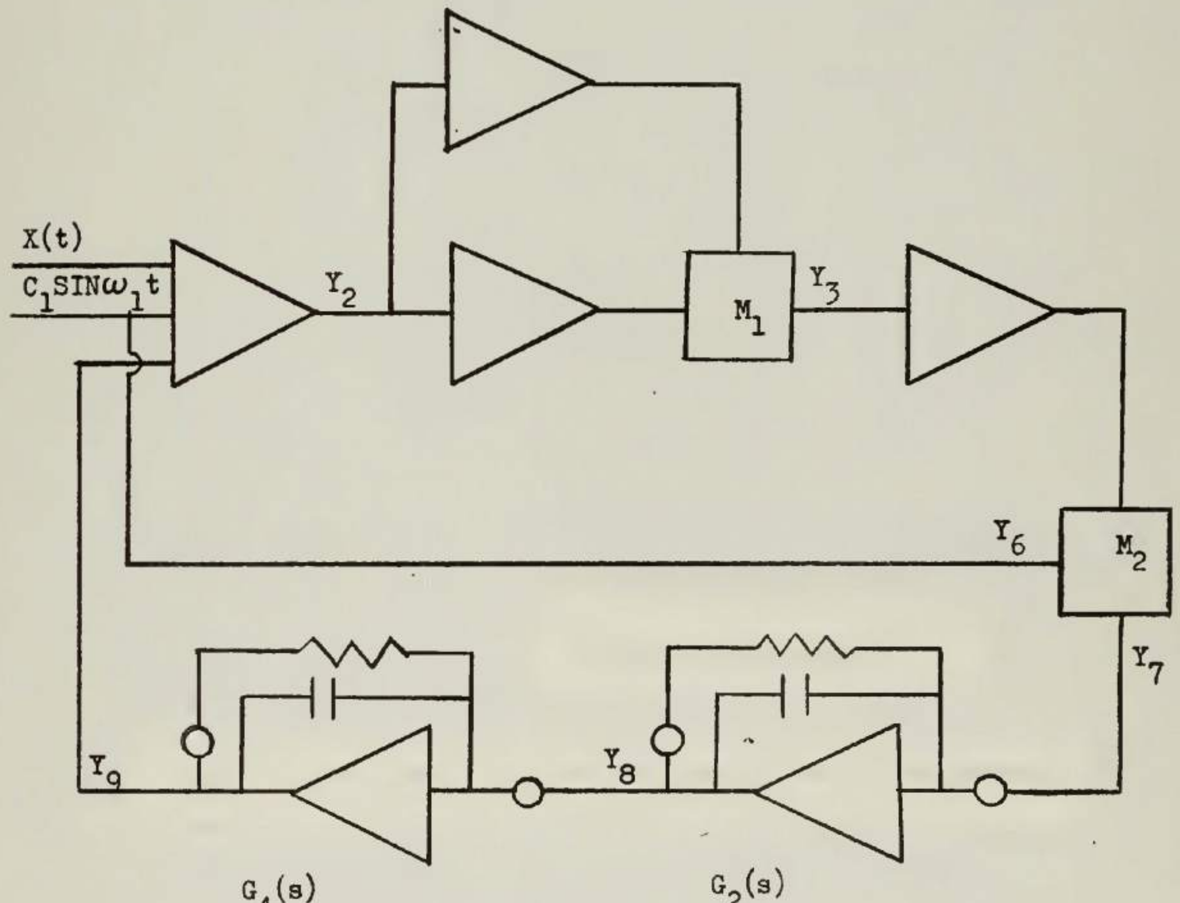
4.3 Adaptive circuit 2

From the simulation study of adaptive circuit 1, it is found that a smoothing filter is needed to eliminate perturbation signal appearing in correction signal. A smoothing filter $G_2(s)$ is inserted between M_2 and the integrator in adaptive circuit 1. The circuit becomes as Fig. 15(a). The analog computer circuit is as Fig. 15(b). This circuit is designated as adaptive circuit 2. The smoothing filter will filter out completely the perturbation signal appearing on the correction signal but also attenuates the correction signal and gives more phase lag to the correction signal. Too much phase lag of the correction signal tends to make a closed loop system unstable. It causes time lag of the correcting parameter and when the parameter change is abrupt such as step change, excessive overshoot results on the correction signal and sometimes it produces a sharp undershoot before producing correction signal in correct direction. The undershoot is a strong correction signal in a wrong direction. It will be studied in more detail later. The smoothing filter is usually chosen as a compensator for the adaptive loop to improve step response of the system.

(a) In the simulation, a filter of the form, $\frac{1}{1 + \tau_2 s}$ is used. The τ_2 values used are 0.2, 0.5, 1, 2, 5, 10. The same perturbation signal and parameter drift signal used in 4.2, are used. Taking the perturbation signal present in correction signal as noise, correction signal to noise ratio is measured at the output point of the smoothing filter $G_2(s)$. Fig. 16 is the graph of correction signal to noise ratio vs. τ_2 of the smoothing filter. According to the graph, τ_2 of 2 or above gives more than 10 : 1 signal to noise ratio. The perturbation waves remaining in the correction signal even after the filtering, will be further attenuated by the integrator following. τ_2 of 2 gives corner frequency of 0.5 rad./sec. and this gives enough



(a) Block diagram $G_4(s)$



(b) Computer circuit

Fig. 15. Adaptive Circuit 2.

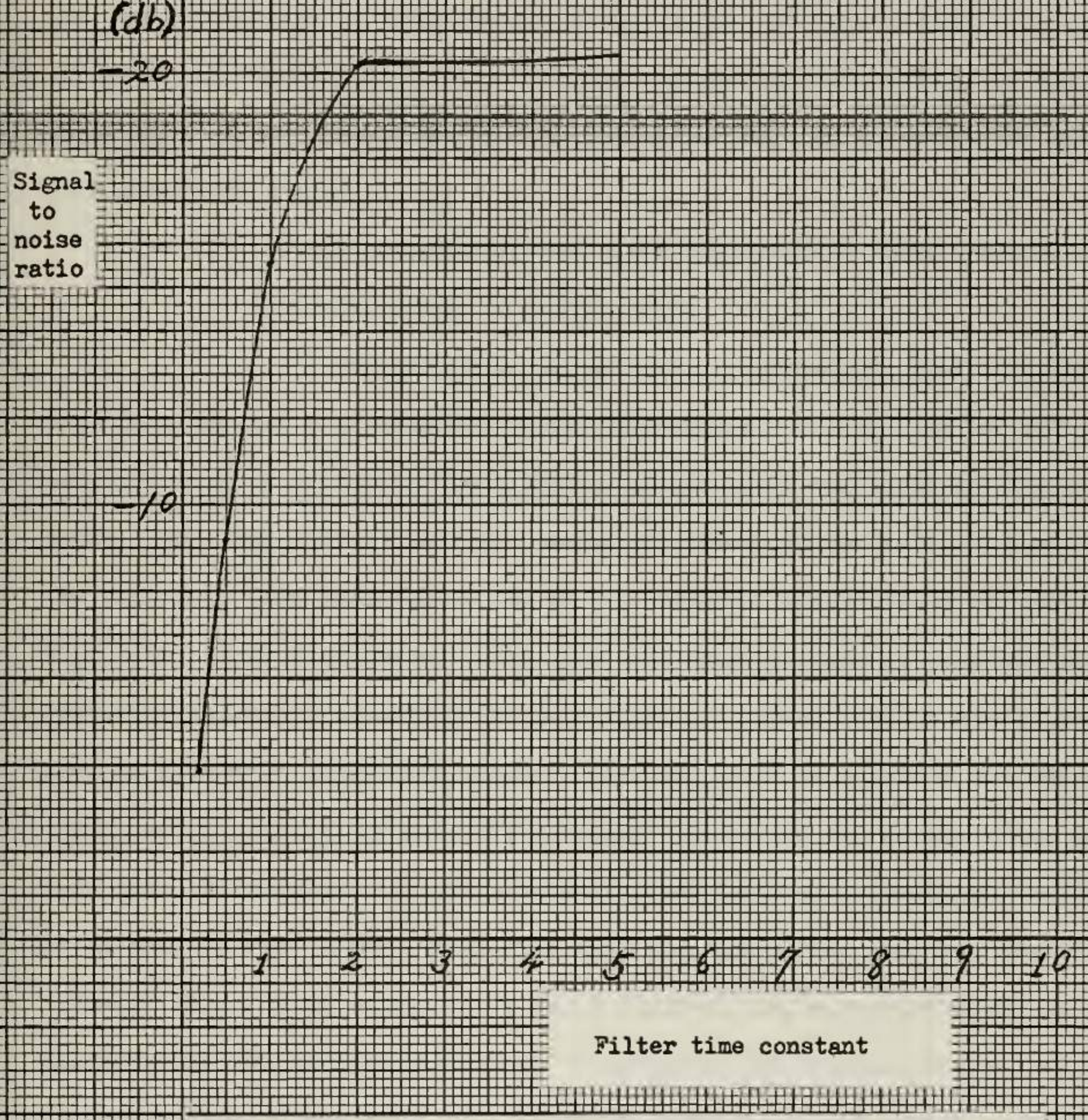


Fig. 16 The signal to noise ratio vs. filter time constant.

attenuation to perturbation waves of 5 cps. present in correction signal of this system. Thus it is concluded that an acceptable ratio of corner frequency of smoothing filter to perturbation frequency is 1 : 60 or greater. But the attenuation of perturbation waves in the correction signal is not so important as the stability of the system, therefore, this filter is designed as an appropriate compensator for the adaptive loop and it naturally acts as a smoothing filter.

(b) Effect of adaptive loop gain on parameter drift correction

(1) Sinusoidal parameter drift

The parameter is drifted sinusoidally. When the adaptive loop is open the parameter drifts sinusoidally and as soon as the adaptive loop is connected the drifting parameter is corrected to the optimum value. The loop gain affects the amount of steady state error as expected. The necessary loop gain is decided by the required steady state accuracy. Fig. 17(a) shows a parameter drift when the adaptive loop is open and then corrected parameter condition when the loop is closed. (b) is the corresponding parameter correction signal. The loop gain is a little low and the steady state error is comparatively large. To reduce the steady state error, loop gain is increased and the parameter drift is almost completely corrected as seen in Fig. 17(c). With this high loop gain, the loop was closed suddenly when the parameter drift was a maximum, the parameter drift was corrected to the optimum value with an excessive oscillation before settling down to steady state; Fig. 18(a) shows the parameter drift being corrected and (b) is the corresponding correction signal wave shape.

(2) Triangular Parameter Drift

Fig. 18(c) and (d) is the triangular parameter drift being corrected and its correction signal. The loop gain is a little low and the

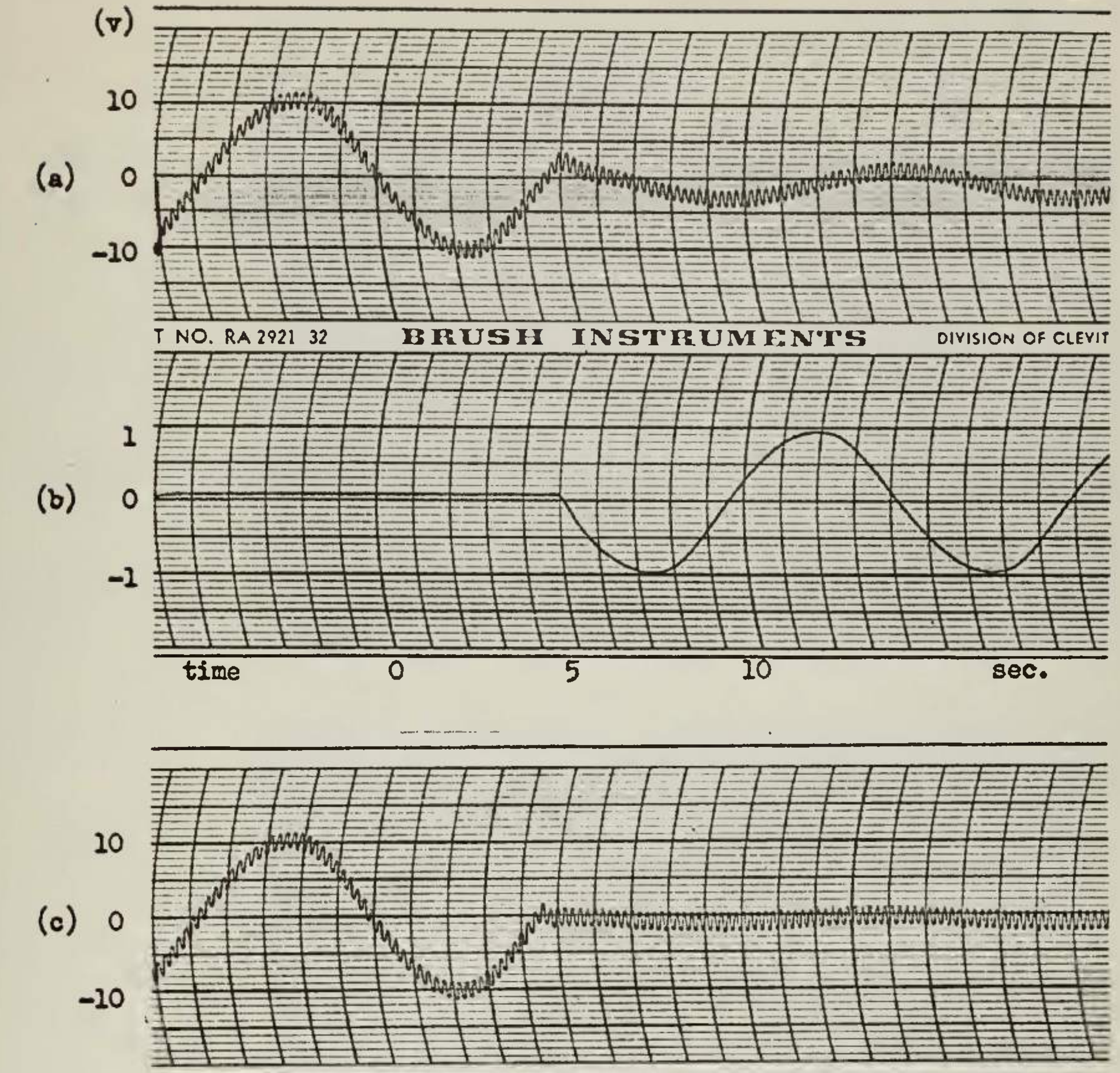


Fig. 17. Parameter drift correction
(a) Parameter drift being corrected with low loop gain
(b) Parameter correction signal
(c) Parameter drift being corrected with a high loop gain

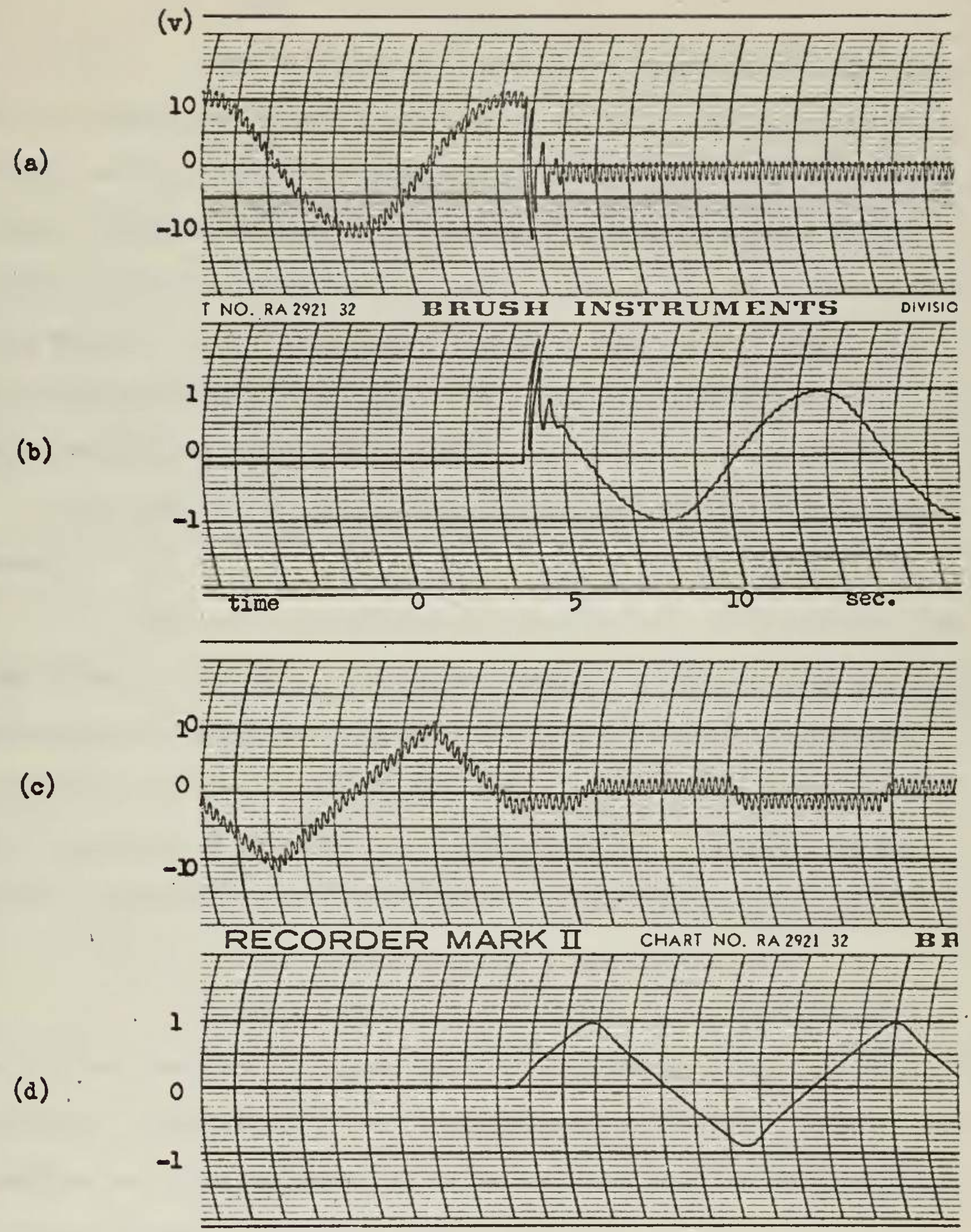


Fig. 18. Parameter drift correction
(a) Sinusoidal parameter drift being corrected with high gain
(b) Correction signal
(c) Triangular parameter drift being corrected
(d) Correction signal

correction is not perfect.

(3) Square wave parameter drift

The parameter is drifted by a square wave. Fig. 19(a) shows the square wave parameter drift being corrected and (b) is the corresponding correcting signal, in which steady state error is small. The system is stable. When the loop gain is too high, the system response becomes oscillatory and tends to be unstable but the system will never be unstable because overall loop is second order system in this set-up. Fig. 19(c) is the parameter being corrected and (d) is the corresponding correction signal when the loop gain is very high.

(c) The effect of perturbation frequency on adaptive loop step response.

The most extreme case of parameter drift is step change of parameter due to an abrupt environmental change. Therefore, only the effect of perturbation frequency on adaptive loop step response is investigated.

The plant parameter is drifted by square wave of 0.01 cps. For different frequencies of perturbation, maximum overshoot and rise time are measured. The perturbation frequencies are varied from 1 cps. to 100 cps.

(1) Steady state magnitude of correction signal

In this adaptive circuit there is no attenuating component for the perturbation signal between perturbation signal input and the demodulation, therefore, the perturbation frequency does not affect the loop gain and the steady state magnitude of correction signal is constant with different perturbation frequency as seen in Fig. 13.

(2) Maximum overshoot and undershoot

For a very low frequency of perturbation, the correction signal has extremely irregular wave shapes of square waves as shown in Fig.

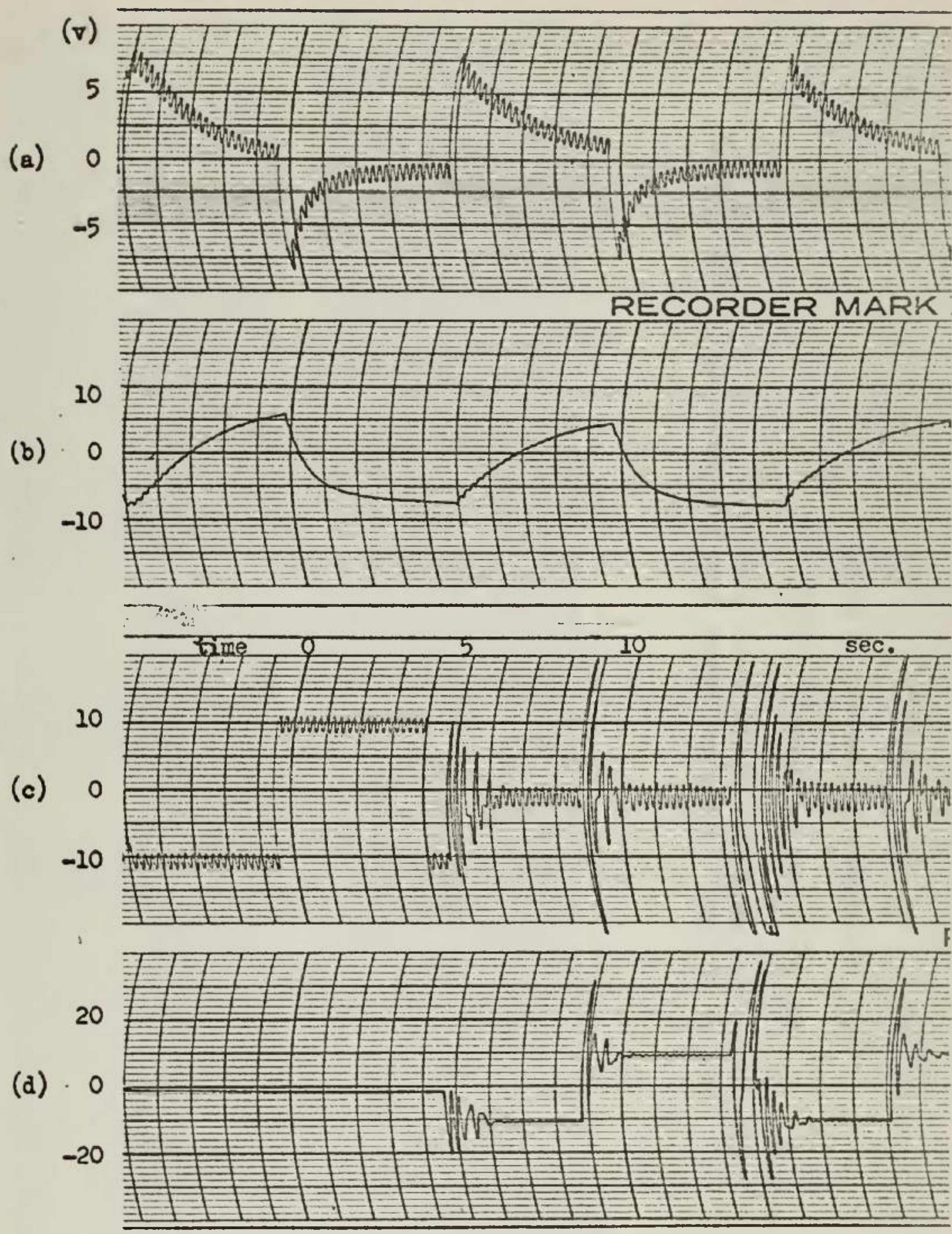


Fig. 19. Parameter drift correction
(a) Square wave parameter drift being corrected
(b) Corresponding correction signal
(c) Square wave parameter drift being corrected with very high loop gain
(d) Corresponding correction signal

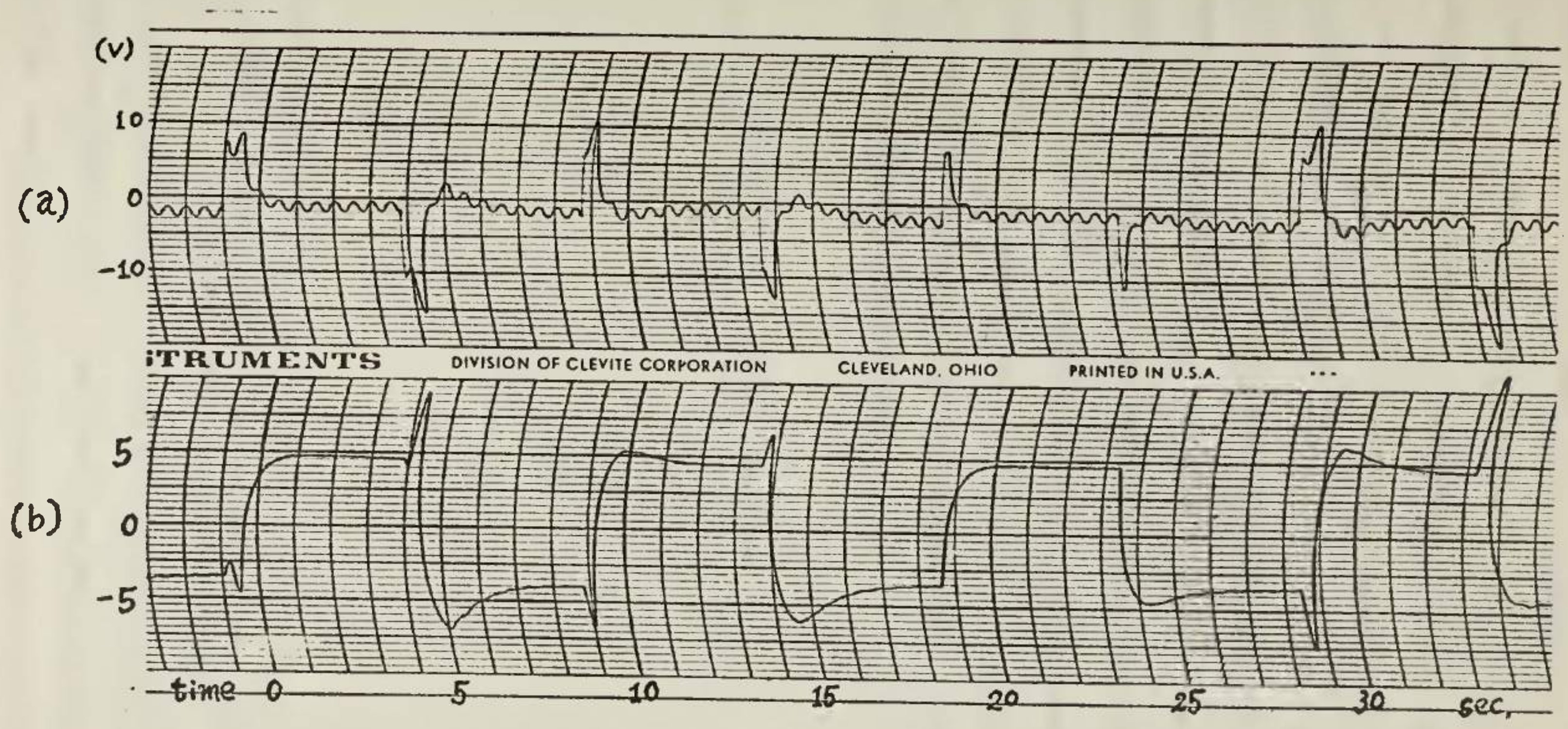


Fig. 20 Parameter drift correction with low frequency of perturbation (perturbation frequency is 2 cps.)

(a) parameter drift being corrected.

(b) corresponding correction signal.

20(b). The perturbation frequency is 2 cps. The correction signal has various amounts of overshoots and undershoots. Here, undershoot is defined as a sharp and strong correction signal to a wrong direction. The magnitude of overshoot and undershoot depends on the point of perturbation signal wave at which an abrupt change of parameter occurs. The irregularities of correction signal wave shapes becomes greater at lower perturbation frequency and with perturbation frequency below 1 cps. the correction signal is too irregular to have a sensible correction. The magnitude of overshoot and undershoot also decreases as the perturbation frequency increases and above 5 cps. of perturbation frequency there is no overshoot nor undershoot. Table 4.1 lists the measured ratio of maximum overshoot and undershoot to steady state correction signal amplitude at lower perturbation frequencies.

ω_1 (cps.)	Max. overshoot	Max. undershoot
1	1.63	4.25
1.5	1.28	2.06
2	1.19	1.81
3	no overshoot	1.38
4	"	1.18
5	"	1.03

Table 4.1 Ratio of Maximum overshoot and undershoot to the amplitude of steady state correction signal.

It is noted from Table 4.1 that the maximum undershoot ratio is much greater than overshoot ratio. The sharp undershoot should be avoided, which makes the system unstable and leads to erratic operation. Therefore, the perturbation frequency should be higher than 5 cps. in this adaptive control system.

(3) Rise time

Rise time vs. perturbation frequency is plotted as Fig. 21. The rise time increases sharply from ω_1 = 1 cps. to ω_1 = 4 cps. Above 5 cps, rise time increases very little and stays almost constant. Fast rise is desirable and it is obtained at lower ω_1 but it accompanies undesirable undershoot as seen in Table 4.1. In the following simulation studies, ω_1 = 5 cps. is used as perturbation frequency throughout, which is the lowest perturbation frequency that does not accompany any overshoot or undershoot in correction signal.

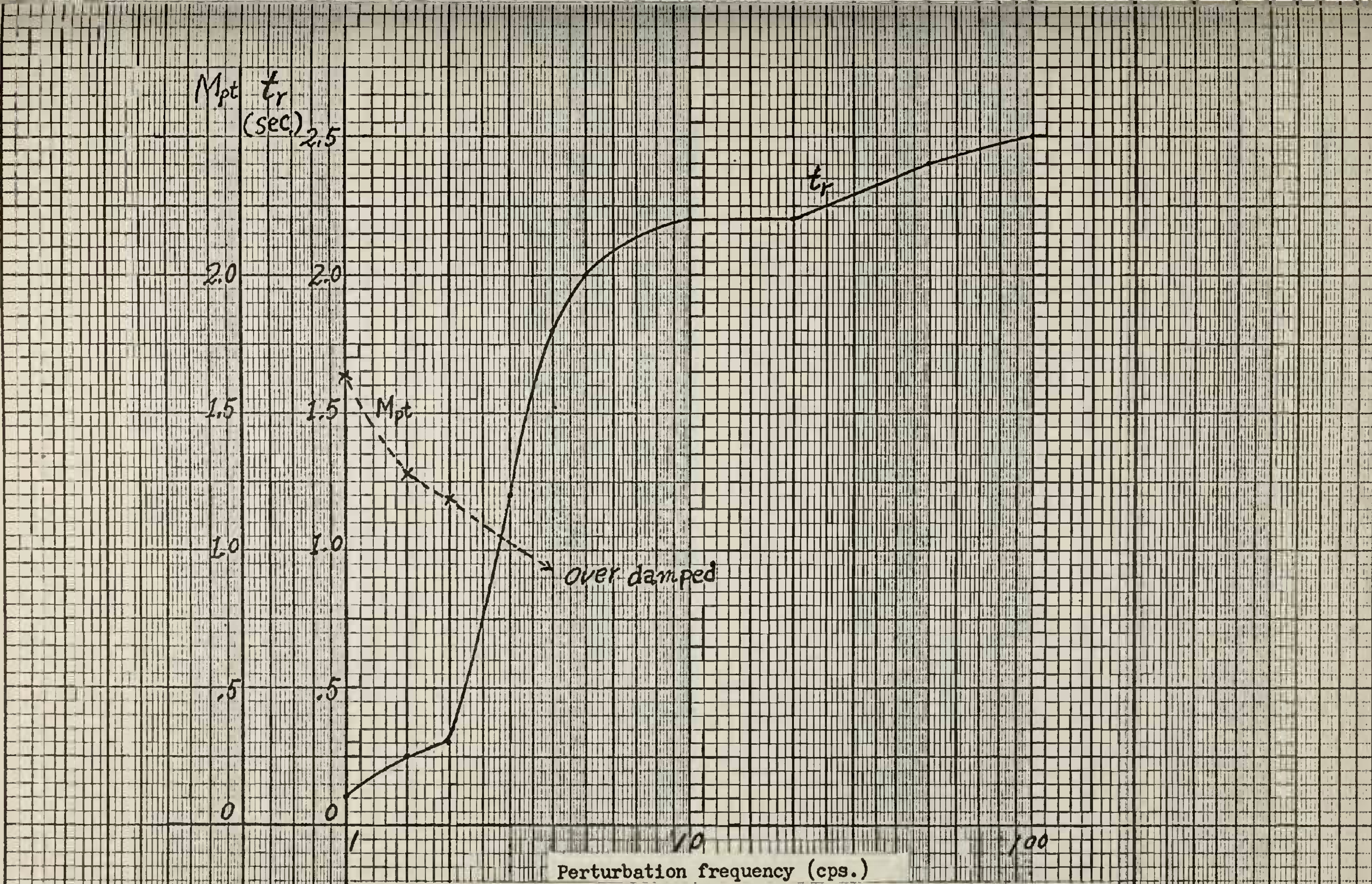


Fig. 21 Maximum overshoot and rise time vs. perturbation frequency

4.4 Adaptive Circuit 3

A band pass filter $G_1(s)$ was added between the squarer and the detector in adaptive circuit 2, Fig. 15. The circuit becomes as shown in Fig. 22. The band pass filter attenuates all unnecessary frequency components considerably in the signal coming from squarer and passes the components which contain information without attenuation. The band pass filter is essential in a multi-loop adaptive control system where different perturbation frequencies carry information for different parameters being controlled through different loops. This filter also affects the stability of the adaptive loop. The requirement of the band pass filter is explained in Section 3.3. To meet the requirement a filter of following form is used,

$$G_1(s) = \frac{\omega_1^2 s}{s^2 + 2\zeta\omega_1 s + \omega_1^2} \tag{4-3}$$

where ω_1 = perturbation frequency (center frequency)

ζ = damping ratio

The half power frequency of this filter is calculated as,

$$\omega_h = \omega_1 + \zeta\omega_1 \tag{4-4}$$

The band width $\omega_{b.w.} = 2\zeta\omega_1$

To decide the ζ value which gives best step response to the system, the ζ is varied and the step response of the system is observed. ζ of 0.5 or 0.6 gives best response.

(a) First, only the band pass filter is simulated. ζ of 0.5 is taken and the filter becomes as,

$$G_1(s) = \frac{\omega_1^2 s}{s^2 + \omega_1 s + \omega_1^2} \tag{4-5}$$

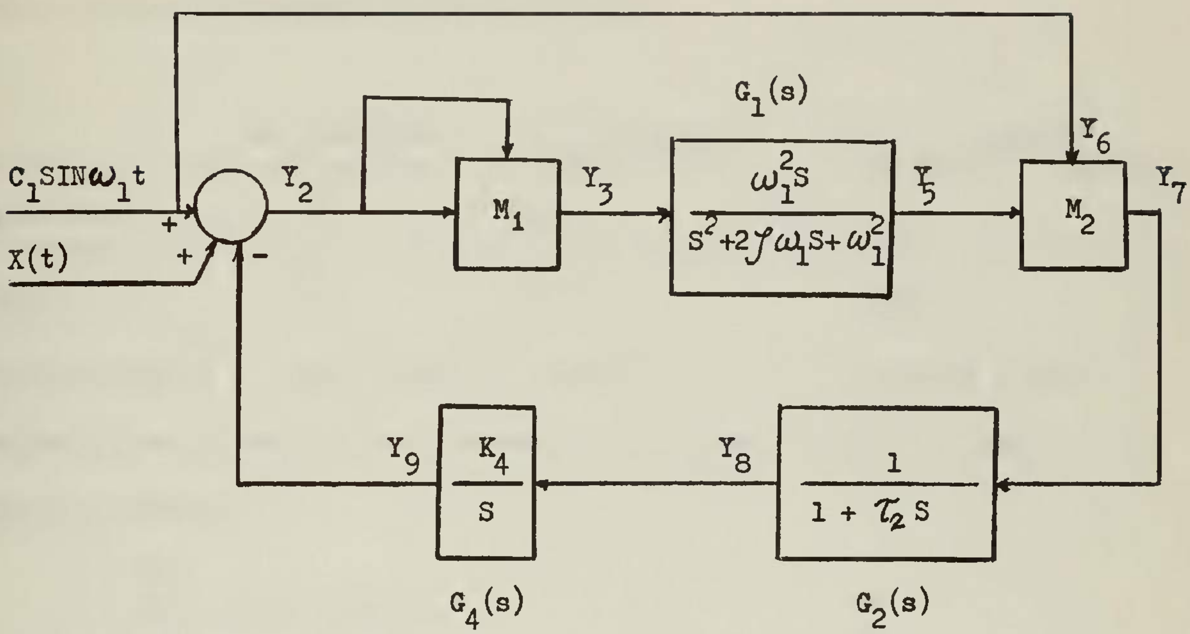


Fig. 22. Adaptive circuit 3

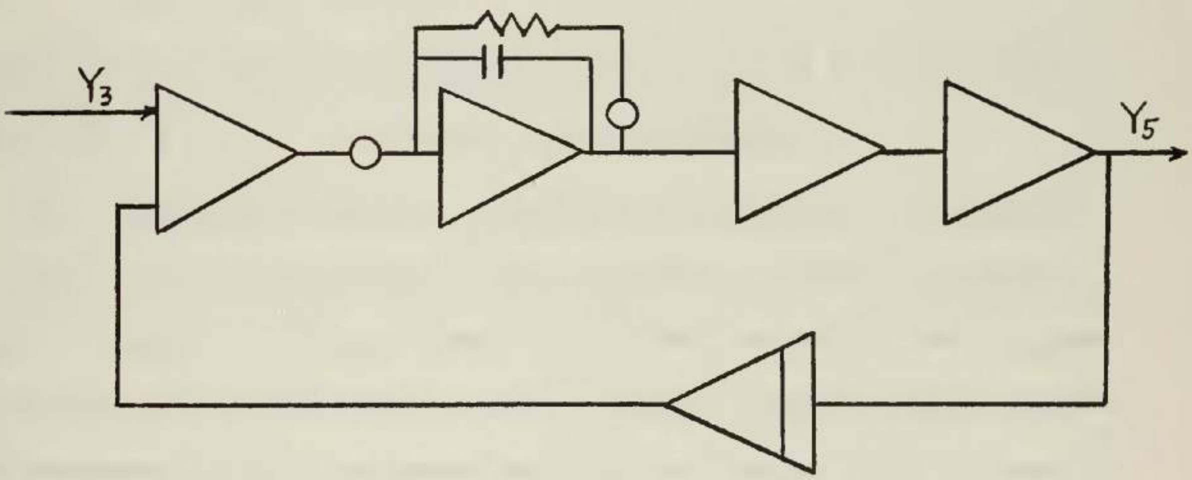


Fig. 23. Analog computer simulation of band pass filter $G_1(s)$

The analog computer simulation of the band pass filter is shown in Fig. 23. The frequency response and phase angle characteristic is plotted as Fig. 24. The calculated half-power frequency and band width and the actual half-power frequency and band width are as below,

	ω_{h1} (rad/sec)	ω_{h2} (rad/sec)	$\omega_{b.w.}$ (rad/sec)
calculated	15.7	47.1	31.4
actual	19.2	49.5	30.4

Using the result of signal analysis in Section 3.3(a), following signal components are expected to be present in the signal from the squarer of adaptive circuit 3.

D.C

$$2\omega_m = 1.256 \text{ rad/sec}$$

$$2\omega_1 = 62.8 \quad ''$$

$$\omega_1 - \omega_m = 30.144 \text{ rad/sec}$$

$$\omega_1 + \omega_m = 32.656 \quad ''$$

Therefore, this filter will suppresses D.C., $2\omega_m$, and $2\omega_1$ and passes $\omega_1 - \omega_m$, and $\omega_1 + \omega_m$ components without attenuation.

(b) The adaptive circuit 3 simulation was obtained by connecting the band pass filter simulation, Fig. 24 between M_1 and M_2 in adaptive circuit 2, Fig. 15. The performance of the adaptive loop is much improved by the addition of the band pass filter. With proper loop gain, a square wave parameter drift is corrected perfectly as shown in Fig. 25, where (a) is the square wave parameter drift being corrected and (b) is its correction signal. The M_{pt} is 1.08 and the rise time is 0.7 sec. Even with excessive loop gain, the performance is less oscillatory compared to the same case in adaptive circuit 2, in 4.3(b)(3). Fig. 25(c) is the parameter drift being

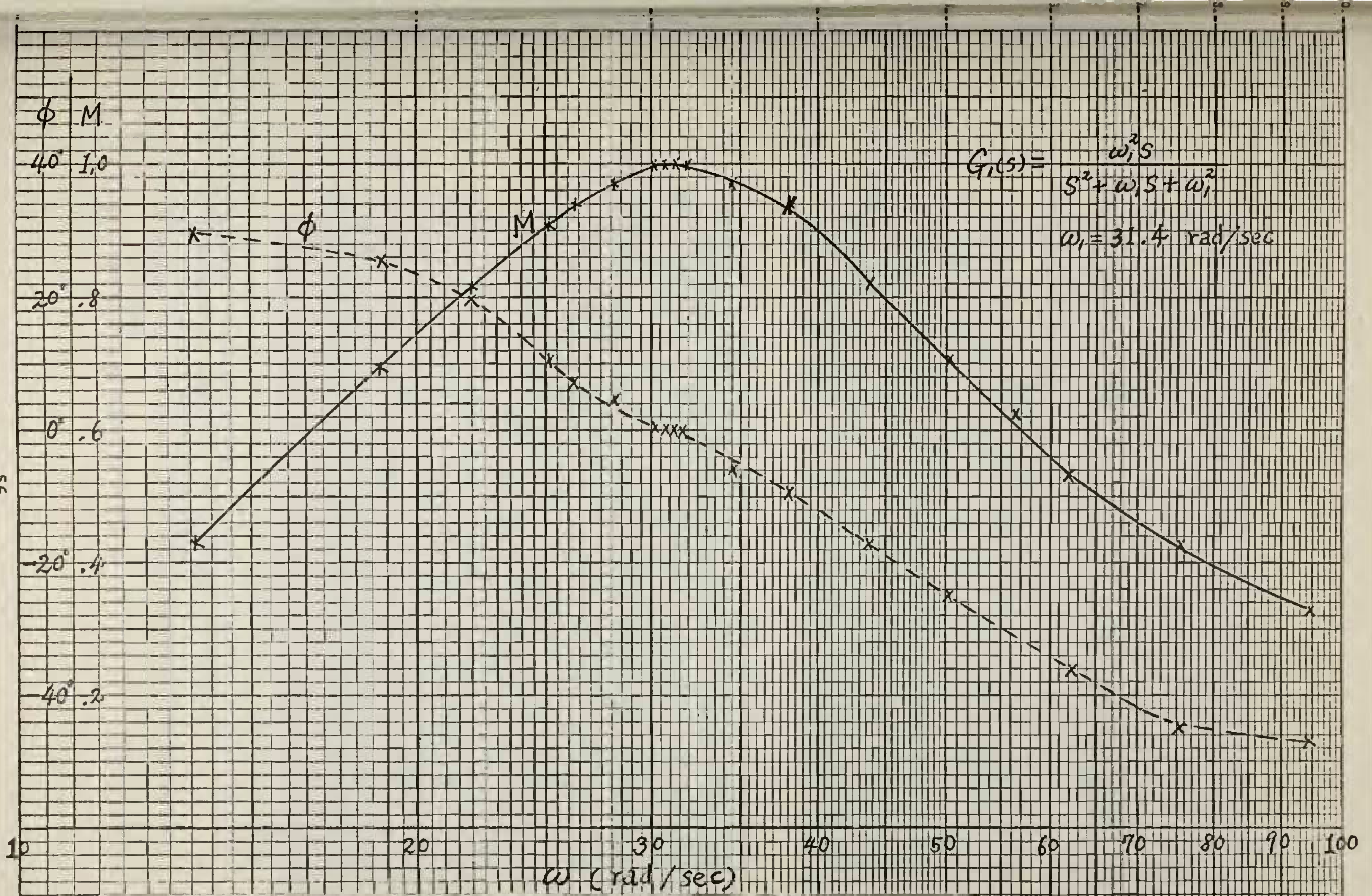


Fig. 24 The frequency response and phase angle characteristic of band pass filter

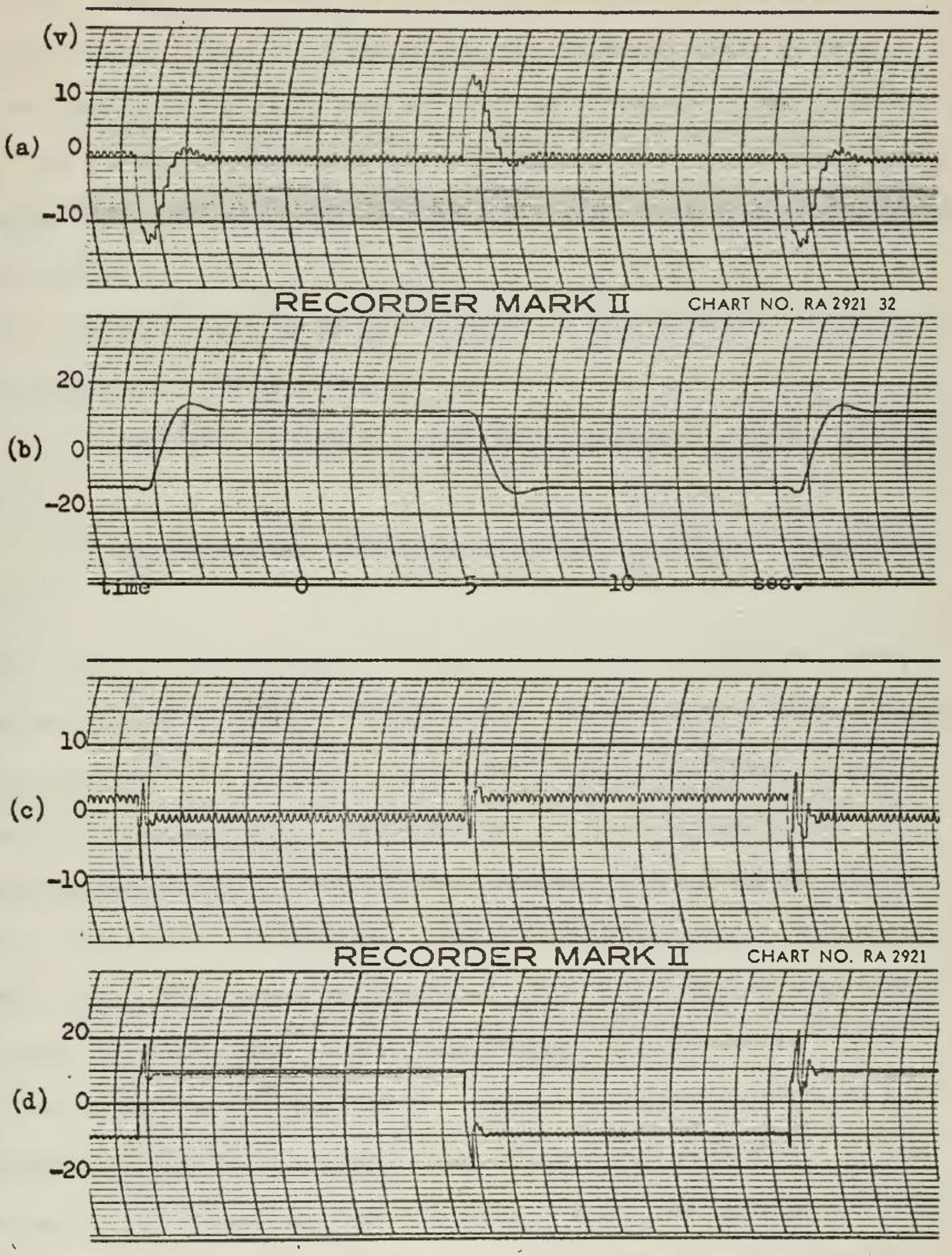


Fig. 25. Parameter drift correction
(a) Square wave parameter drift being corrected
(b) Corresponding correction signal
(c) Square wave parameter drift being corrected with excessive loop gain
(d) Corresponding correction signal

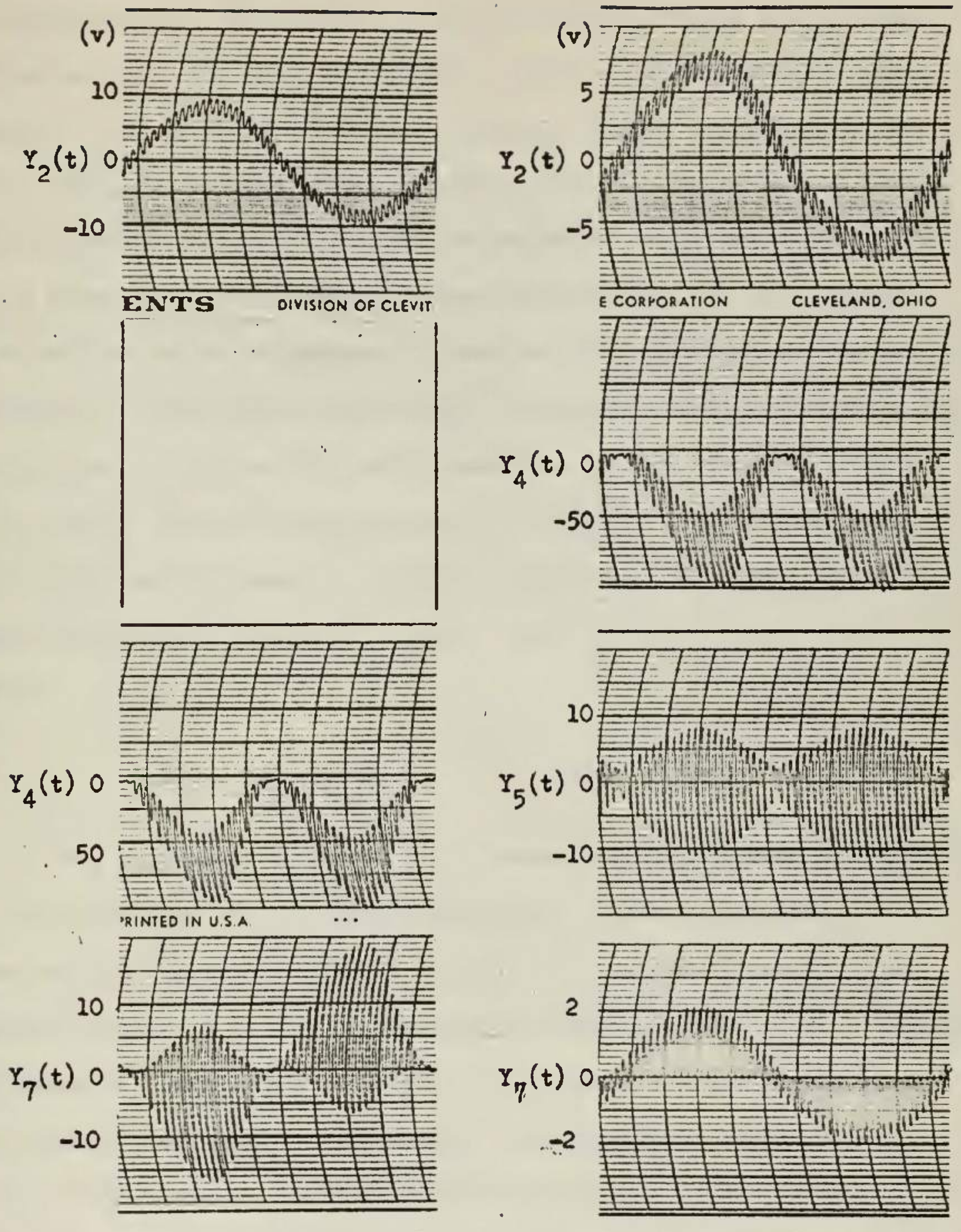
corrected with excessive loop gain and (d) is its correction signal. Compare these figures with Fig. 19(c) and (d) which are the same case in adaptive circuit 2. Fig. 25(c) shows that the parameter drift is over-corrected because of excessive loop gain but the response is less oscillatory. The comparison shows a definite improvement in adaptive loop performance by adding band pass filter. This filter is acting as a compensator in this circuit.

(c) A comparison of demodulated outputs is made between following two cases in sinusoidal parameter drift.

(1) Modulated wave is not filtered by band pass filter. (adaptive circuit 2).

(2) Modulated wave is filtered by band pass filter. (adaptive circuit 3).

Fig. 26(a) is for the case 1. Refer to the Fig. 22 adaptive circuit 3 for the designation of signals at each point. For the case 1 the modulated signal Y_3 goes into the detector without filtering by band pass filter. The demodulated output is Y_7. The modulated signal Y_3 contains many unnecessary signal components which do not contain any intelligence as explained in section 3.2. Accordingly, the demodulated output contains unnecessary signal components. On the other hand, in the case 2, Fig. 26(b), the Y_3 passes the band pass filter which filters out unnecessary signal components as explained in section 3.3. Thus, the signal Y_5 contains only signals which contain intelligence. Note the wave shapes after filtering. This signal is demodulated and comes out as Y_7 which has clear cut polarity in accordance with the parameter drift direction. This indicates the Y_7 for the case 2 does not contain any unnecessary signal as in the case 1. This verifies the signal analysis in section 3.3 and 3.4.



(a) (b)

Fig. 26. Comparison of demodulated output with modulated signal filtered and unfiltered.
(a) Unfiltered (b) Filtered

4.5 Adaptive circuit 4

The measurement lag filter $G_m(s)$ is connected between the parameter perturbation and drift input and squarer. The detection reference phase lag filter $G_3(s)$ is connected between reference signal input and the detector. The $G_m(s)$ represents the phase lag in measuring error. $G_3(s)$ is chosen to give the same phase lag to the detection reference signal at ω_1 as $G_m(s)$ gives to the perturbation signal (the carrier) at ω_1, thus the carrier and the detection reference signal are in phase with each other at the detector. The band pass filter $G_1(s)$ does not introduce any phase lag to the carrier at ω_1, and the total phase lag introduced to carrier between Y_1 and Y_5, is the only phase lag due to $G_m(s)$. The circuit becomes as Fig. 27 and analog computer simulation is as Fig. 28. This circuit is designated as adaptive circuit 4. The filters $G_m(s)$ and $G_3(s)$ take following forms,

$$G_m(s) = \frac{1}{1 + \tau_m S}, \qquad G_3(s) \quad \frac{1}{1 + \tau_3 S}$$

(a) The process of demodulation is explained and analyzed in section 3.4. It is verified in this simulation study. First, $\tau_m = \tau_3 = \frac{1}{31.4}$ is used, which gives 45° phase lag at ω_1 = 31.4 rad/sec/(5cps.). Thus the carrier signal in Y_5 and the detection reference signal Y_6 are in phase at the detector. The modulated signal Y_5 is multiplied by the Y_6 signal and the demodulated output is produced. The demodulated output has one polarity or the other in accordance with the parameter drift direction with respect to the optimum parameter value. Fig. 29(a) is the modulated signal Y_5 and (b) is the demodulated output Y_7. The demodulated output has exactly the same wave shape as obtained by graphical multiplication of Y_5 and Y_6 in

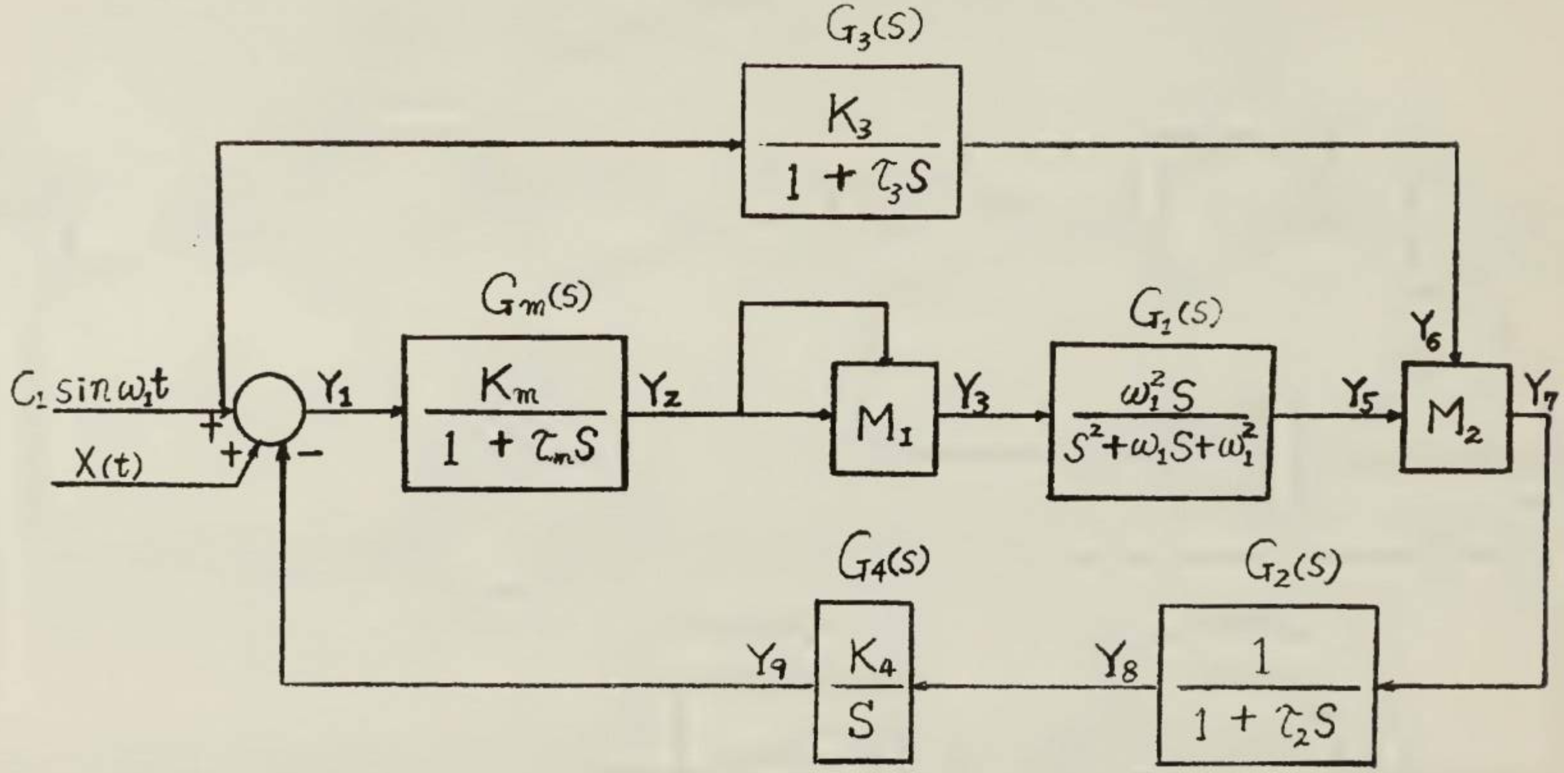


Fig. 27 Adaptive circuit 4.

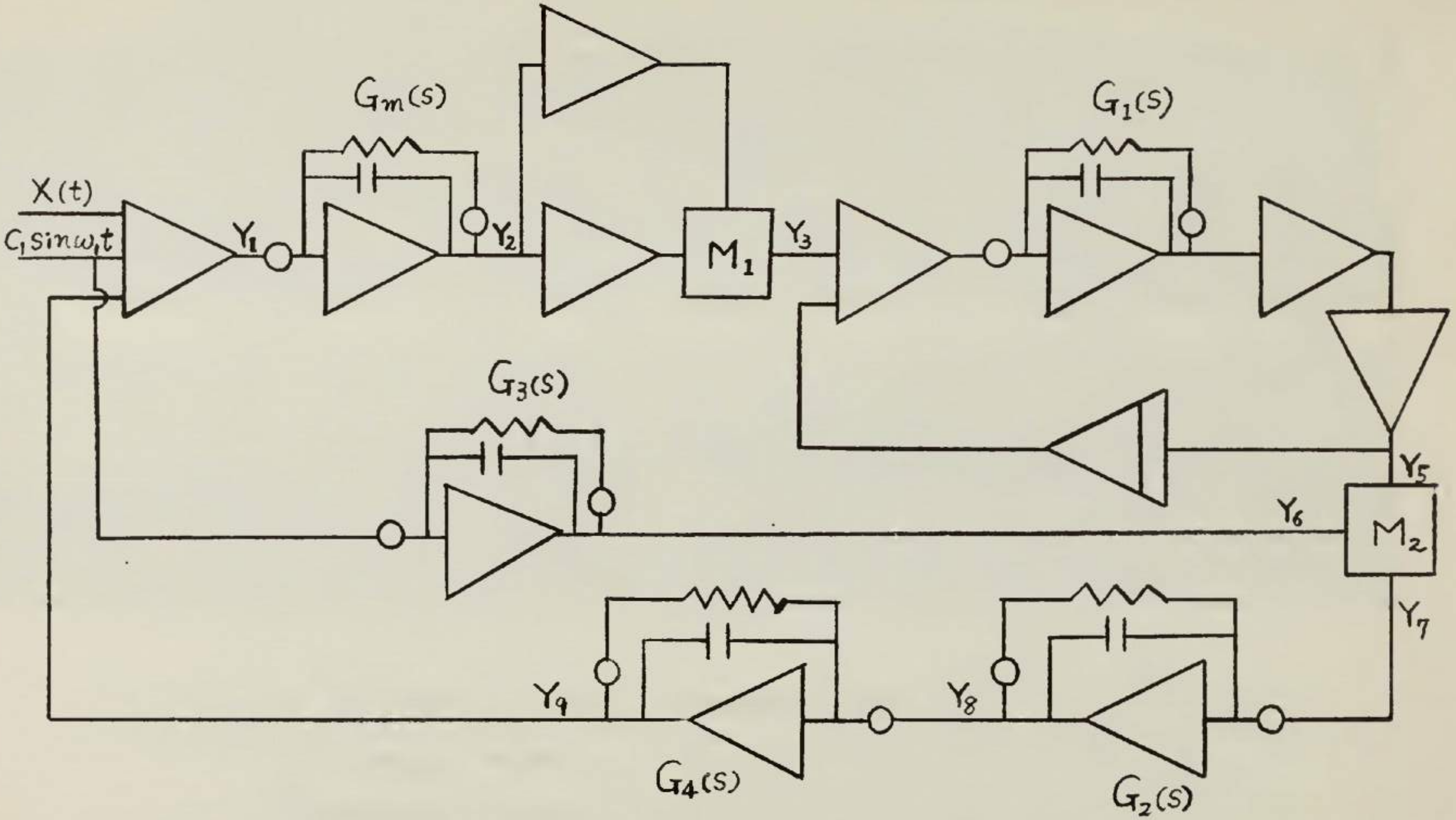


Fig. 28 Analog computer circuit for the adaptive circuit 4.

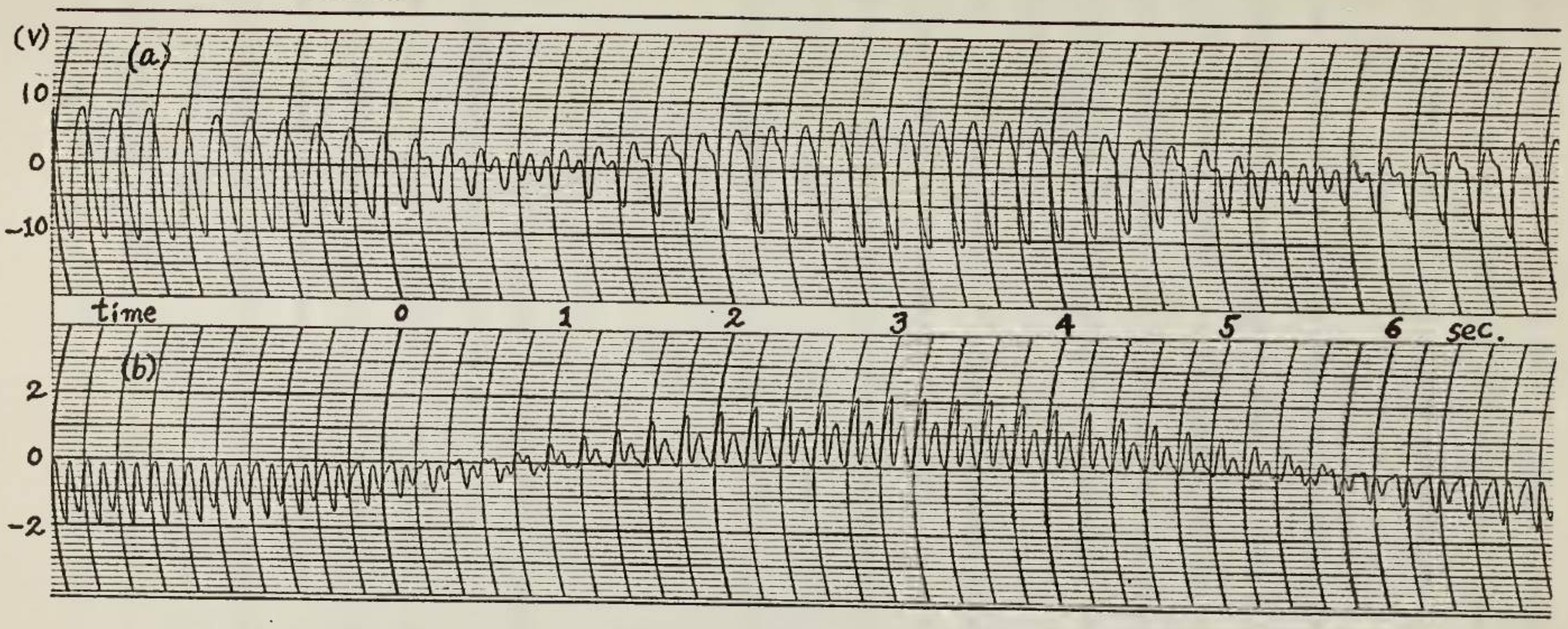


Fig. 29 The modulated signal and demodulated output.

(a) modulated signal, Y_5

(b) demodulated output, Y_7

section 3.4 Fig. 5. This signal is integrated and develops a correction signal of one polarity or the other. When the polarity of detection reference signal Y_6 is changed, the demodulated output also changes its polarity. These results confirm the explanation of the demodulation in section 3.4.

The modulated signal and demodulated output wave shapes for square wave parameter drift are shown in Fig. 30. The (a) in this figure is parameter drift, (b) is the output of squarer, (c) is the modulated signal coming out of the band pass filter and (d) is the demodulated output. The demodulated output has one polarity or the other depending on the direction of parameter drift.

(b) Next, the effect of phase difference between Y_5 and Y_6 on the demodulated output is observed. This effect is explained in section 3.4 and the result of graphical multiplication of two signals which are out of phase with each other by some angle is shown in Fig. 5. The response of the system is observed for the following three cases.

1. Y_6 and Y_5 are in phase.
2. Y_6 leads Y_5 by 60°.
3. Y_6 lags Y_5 by 60°.

For the case one, the parameter drift being corrected and parameter correction signal wave shapes are as Fig. 31(a) and (b) respectively; the M_{pt} is 1.046 and rise time is 0.9 sec. For the case 2 and 3, the results are the same and the result of the case 2 is shown in Fig. 31(c) and (d). The response is overdamped and the rise time is about 2 sec. This shows that the overall loop gain is reduced due to the phase difference between the Y_5 and Y_6. This effect is predicted in the signal analysis in section 3.4. The phase difference between Y_5 and Y_6 does not affect the system seriously and merely reduces the loop gain which can be easily restored.

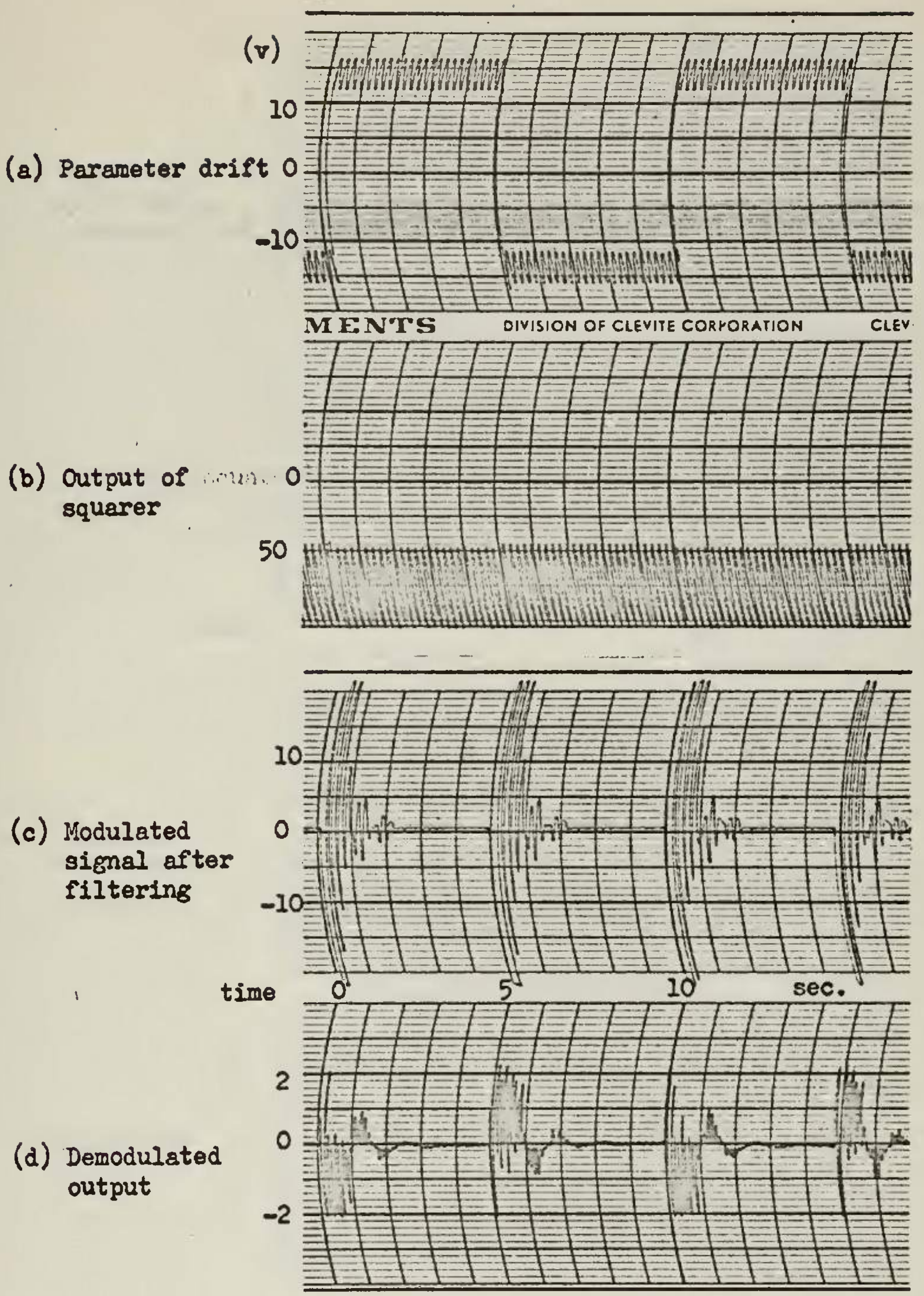


Fig. 30. Modulation and demodulation of square wave parameter drift.

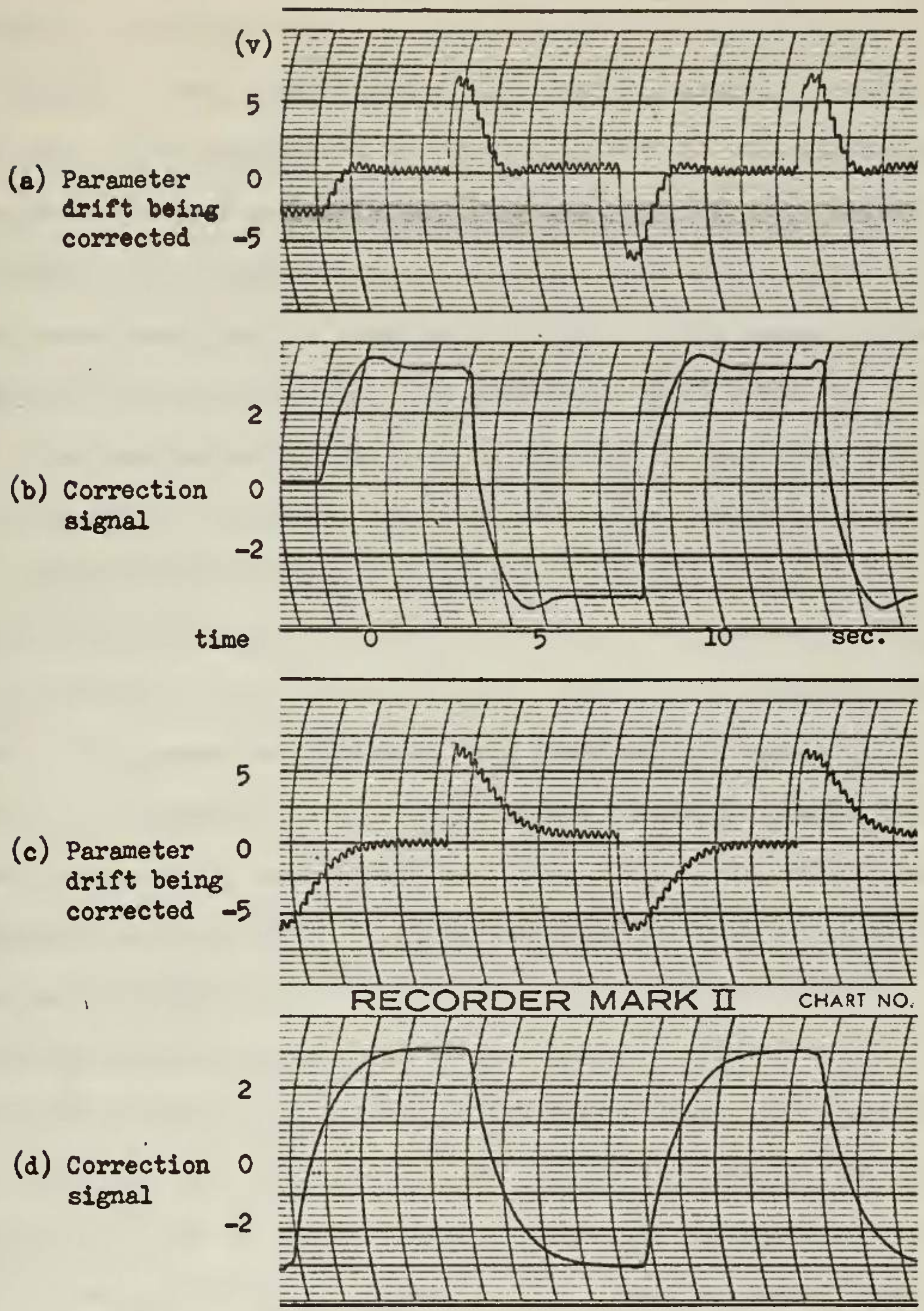


Fig. 31. Effect of phase difference between Y_5 and Y_6
(a),(b) Y_6 and Y_5 are in phase.
(c),(d) Y_6 leads or lags Y_5 by 60°.

4.6 Complete adaptive control circuit (adaptive circuit 5)

Finally, a complete adaptive control circuit was simulated on an analog computer. The block diagram of the complete system is as Fig. 32. This circuit will be designated as adaptive circuit 5. The analog computer simulation is achieved by merely replacing the $G_m(s)$ in the adaptive circuit 4 simulation, Fig. 28 with the model and plant simulation circuit in Fig. 7 and the correction signal is negatively fed back to the summer, output of which goes to the multiplier M_3. The reference signal phase lag filter $G_3(s)$ is not used because the carrier is expected to have phase shift of 160° at the detector, which is nearer to 180°. The input to the plant and model, is $R(t) = 7\sin 1.884t$ ($f = 0.3$ cps.).

(a) The smoothing filter $G_2(s)$ is acting as a compensator. To see the effect of the $G_2(s)$ on the system response, first it was removed and the response of the system was observed. The adaptive loop was opened at the output of the integrator $G_4(s)$. The parameter correction signal developed with the loop open, is recorded as Fig. 33(b). Fig. 33(a) is the square wave parameter drift signal. To close the adaptive loop, the polarity of the correction signal should be reversed to have negative feedback.

When the loop is closed, this system becomes unstable, so the loop gain was reduced to a very small value such that the system barely maintains a stable condition. The correction signal becomes as shown in Fig. 33(c). The magnitude of the correction signal becomes very small and the signal shape is drastically changed compared to the correction signal shape with the loop open. That is why the open loop signal analysis is not exactly valid when the loop is closed as mentioned in section 3. Fig. 33(d) is the plant output and (e) is the model output. Because of too small loop gain and accordingly too small correction signal, a slight correction of

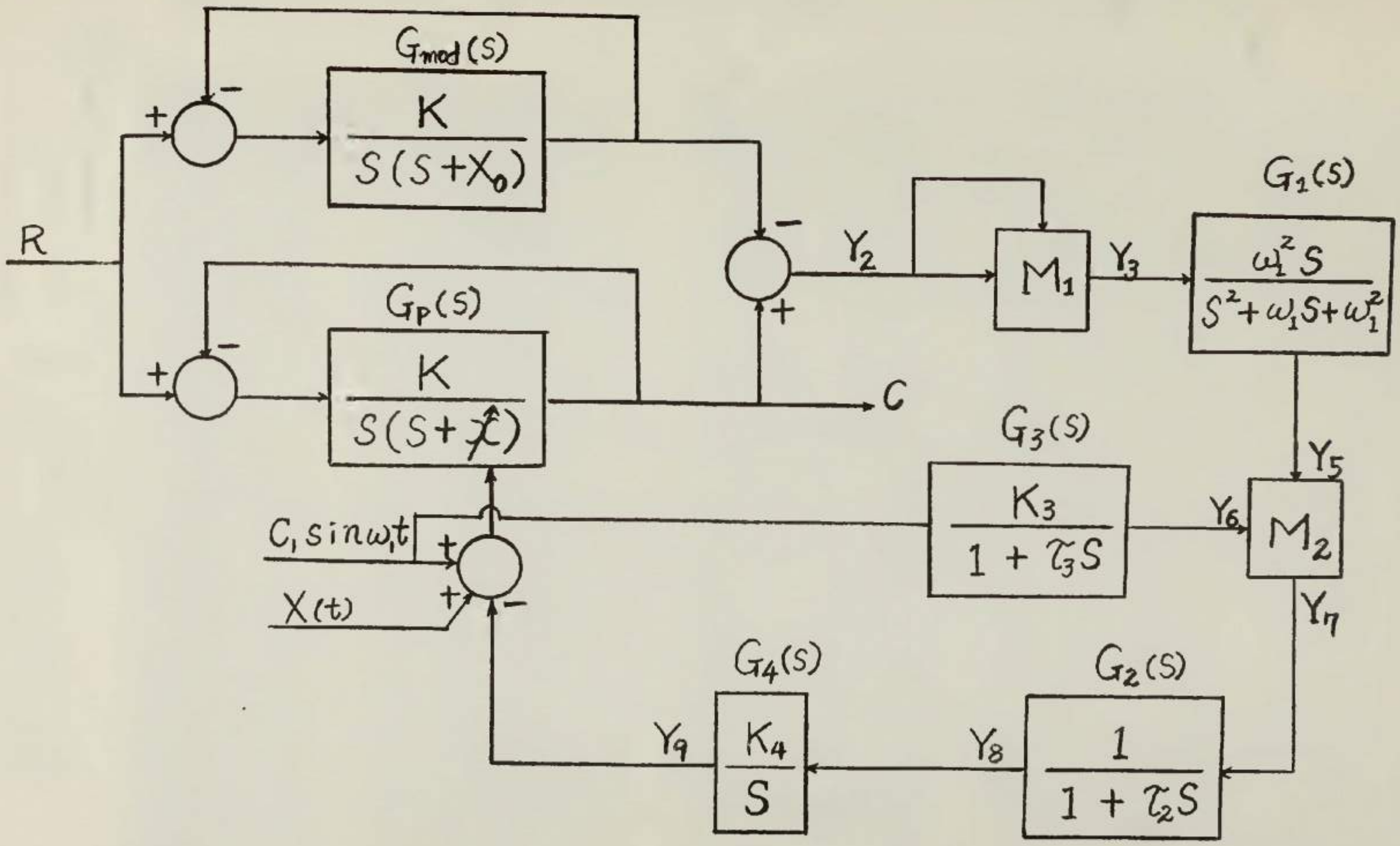


Fig. 32 The complete adaptive control circuit (Adaptive circuit 5)

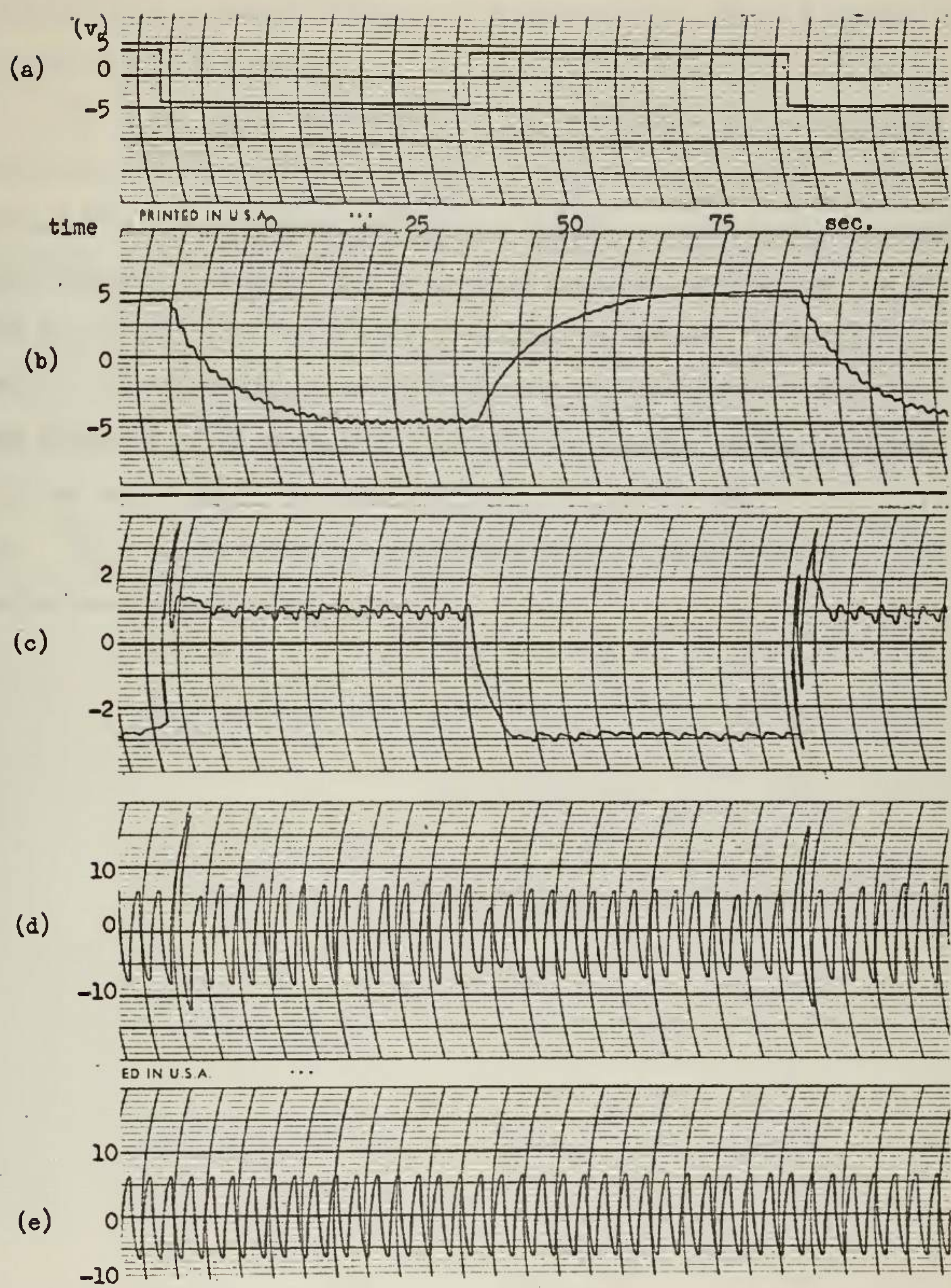


Fig. 33 Signal shapes in adaptive circuit 5.

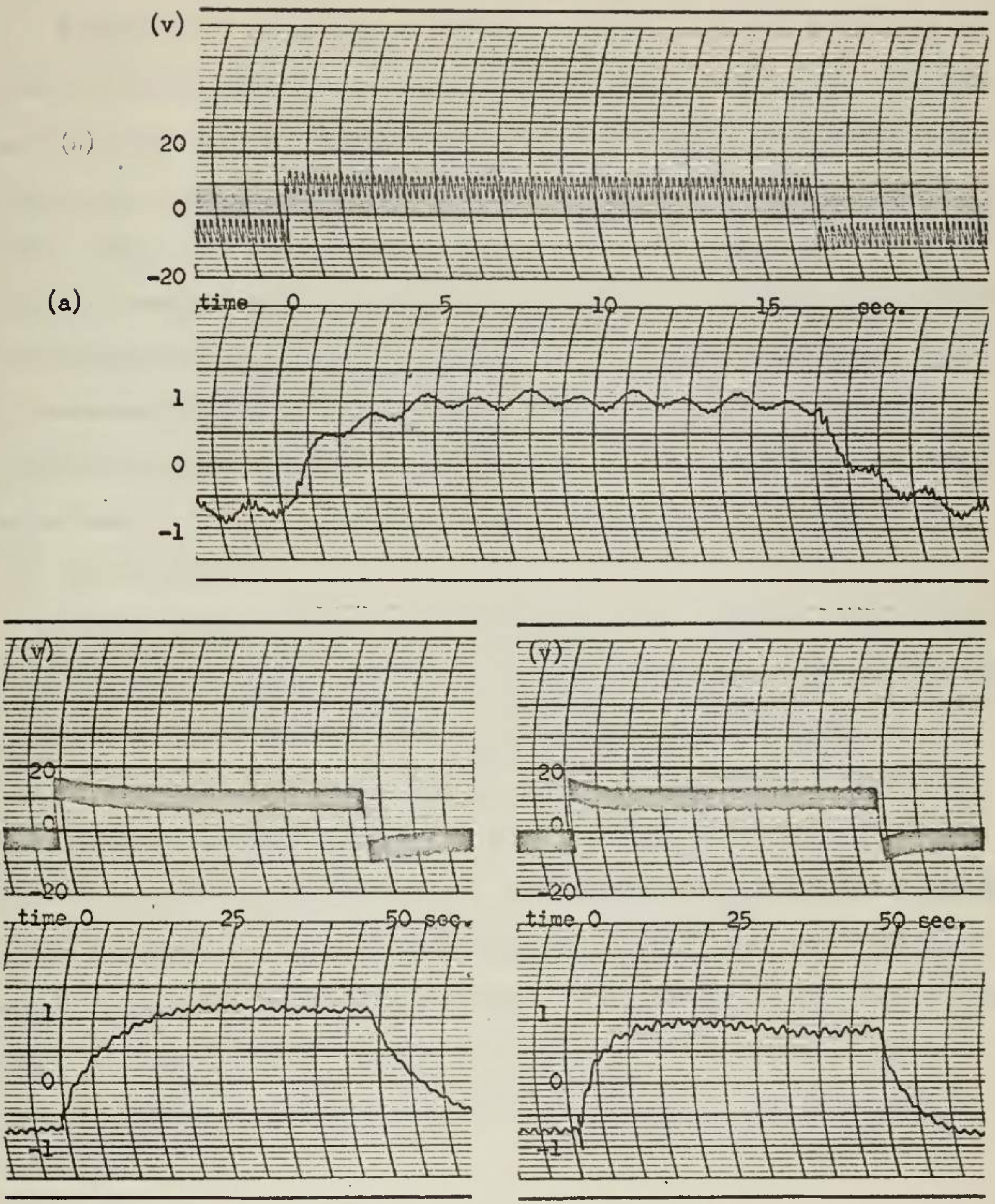
(a) parameter drift. (b) Correction signal (open loop)

(c) correction signal (closed loop). (d) plant output

(e) model output.

parameter drift is achieved. Compare the figures (d) and (e). When the plant parameter changes abruptly the plant output has sharp overshoot due to instability of the system.

(b) A lag filter of the form $G_2(s) = \dfrac{1}{1 + \tau_2 s}$ is inserted between the detector and the integrator. Three different values of τ_2, 2, 5, and 10 are used and the loop gain is adjusted to give stable operation. With different values of τ_2, the correction signal wave shapes are different but in all cases the critical loop gain is too low. Thus the parameter drift can not be corrected completely in this circuit. Fig. 34 is the parameter drift being corrected and corresponding correction signal for the three cases; Fig 34(a) is for $\tau_2 = 2$, (b) is for $\tau_2 = 10$, (c) is for $\tau_2 = 5$. An effective compensator must be found to achieve a required performance of the system.



(b) (c)

Fig. 34 Parameter drift and correction signal
(a) $\tau_2 = 2$ (b) $\tau_2 = 10$ (c) $\tau_2 = 5$

5. Design of a Sinusoidal Perturbation Adaptive Control System

A design of a sinusoidal perturbation system is attempted based on the simulation study and using a linearization technique. The design procedure and considerations are virtually the same for this system as general control systems except some special features such as modulation and demodulation of error signal using perturbation signal. Finding the approximate equivalent transfer function of circuit components and applying linearization technique, a preliminary design is made on analytical basis and the preliminary design is simulated on analog computer and the effectiveness of the analytical method is checked. The final refinement of design should be done on experimental basis.

5.1 Specification

There are many ways to specify the performance criteria. Those specifications differ depending on the application, but at least the steady state requirement and dynamic or transient requirement should be specified. One special specification for this system is hunting loss, which is defined as the maximum allowable deviation of parameter caused by sinusoidal perturbation at the optimum parameter value. The choice of the hunting loss depends on the requirement of the system and the characteristic of the IP curve; a small hunting loss gives less distrubance on the output of the plant but may make the system lock into small local minimum in IP curve and sometimes does not give any information about the actual parameter drift because of attenuation.

5.2 Selection of Perturbation frequency

The perturbation frequency ω_1 should be much higher than the expected maximum frequency of the parameter drift ω_m; this is necessary in any modulation system. The investigation of the effect of perturbation frequency

on the system response in 4.3(b) shows that a perturbation frequency of higher than 5 cps, does not make much difference on the system response. At an extremely low frequency of perturbation, the system operation tends to become erratic due to sharp overshoot and undershoot even though the rise time is very short. If the perturbation frequency is too high, the attenuation through the system components is severe and the higher gain of perturbation is required.

5.3 Linearization of the System

To investigate the stability and response of the system analytically, an equivalent signal transfer function of the adaptive loop is established. The representative circuit block diagram is derived in section 3.1 and shown in Fig. 4. The following linearization technique is applied to linearize the adaptive loop. In the adaptive loop, a D.C. parameter correction signal is proportional to the plant parameter error $X - X_0$, as shown in equation (3-31) section 3.4(b). This indicates a possibility of derivation of an equivalent linear loop. In applying the linearizing technique to any specific system, only the variation of the IP curve about the optimum is important. Therefore in the general IP curve equation, set $X_0 = 0$ and $a_2 = 0$. Referring to the Fig. 27 adaptive circuit 4, which is a complete block diagram of the system, the portion between error output and the detector is a carrier system. To linearize the squarer and the detector, the describing function or a first term of Taylor series approximation is used and to get an equivalent signal transfer function for band pass filter $G_1(s)$ frequency transformation technique is used. (Ref. 1)

The perturbation signal does not get into the picture when a closed loop analysis is made as verified in simulation study 4.2. Therefore, only the

modulating signal (parameter drift signal) is acting in the closed loop.

(a) The parameter drift signal acts directly on the $G_m(s)$ and no linearization is necessary for $G_m(s)$.

(b) Linearization of M_1

Standard describing function technique is used. The input to M_1 is the error output $Y_2(t)$ and can be rewritten from equation (3-7) as,

$$Y_2(t) = A_p C_1 \sin(\omega_1 t - \theta_p) + A_m X_1 \sin(\omega_m t - \theta_m) \tag{5-1}$$

The output from M_2 is $Y_4(t)$ equation (3-10). In this equation only the last term is the carrier modulated by the intelligence signal and all other terms does not contain any intelligence and will be suppressed by the band pass filter. Therefore, as far as the modulating signal is concerned, only the last term of the $Y_4(t)$ is the output from M_2. The last term of the $Y_4(t)$ is written from equation (3-10),

$$2a_1 A_m A_p C_1 X_1 \sin(\omega_1 t - \theta_p) \sin(\omega_m t - \theta_m) \tag{5-2}$$

The equivalent multiplier gain is the ratio of the peak amplitudes of signal input and output terms. Thus, the peak amplitude of (5-2) is divided by the peak amplitudeof the modulating signal term in (5-1) and one obtains

$$G_{M1} = 2a_1 A_p C_1 \tag{5-3}$$

This same gain also can be obtained by a Taylor series approximation.

(c) Linearized equivalent transfer function of band pass filter $G_1(s)$.

$$G_1(s) = \frac{\omega_1^2 s}{s^2 + \omega_1 s + \omega_1^2}$$

The steady state phase shift due to $G_1(s)$ at $S = j\omega_1$ is 0°.

The linearization is based on the following assumptions;

1. Phase shift curve is linear at $S = j\omega_1$.
2. Attenuation at $S = j(\omega_1 \pm \omega_m)$ is almost the same as at $S = j\omega_1$.

Considering only the modulating signal component, the input to the band pass filter is the signal (5-2) and the output is the last term signal in $Y_5(t)$ equation (3-21). The output is written as,

$$2a_1A_mA_pC_1X_1A_3\sin(\omega_1 t - \theta_p)\sin(\omega_m t - \theta_m - \Delta\theta) \qquad (5\text{-}4)$$

The input (5-2) is compared with the output (5-4), the modulating signal is attenuated by the factor of A_3 and phase lag of $\Delta\theta$ is introduced by the $G_1(s)$. An equivalent transfer function may now be defined such as to approximate the phase characteristics of the original network or,

$$G_{1\text{ equiv.}}(s) = \frac{A}{1 + \tau_{1\text{ eq.}}S} \qquad (5\text{-}5)$$

In this case, A is A_3 which is the attenuation factor of the band pass filter at ω_1. and $\tau_{1\text{ eq.}}$ is determined by choosing a representative value of ω_m and $\tau_{1\text{ eq.}}$ is calculated such that $G_{1\text{equiv.}}(s)$ gives the same phase lag at ω_m as $\Delta\theta$, which results in the original system.

(d) Equivalent transfer function for M_2.

The detection reference signal input to M_2 is

$$Y_6(t) = BC_1\sin(\omega_1 t - \theta_p) \qquad (5\text{-}6)$$

and the modulated signal input to M_2 is the signal term (5-4).

The output of M_2 is the $Y_{7d.c.}(t)$ equation (3-27) and is rewritten as,

$$Y_{7d.c.}(t) = \frac{1}{2}\,(2a_1A_mA_pC_1X_1A_3)BC_1\sin(\omega_m t - \theta_m - \Delta\theta) \qquad (5\text{-}7)$$

The ratio of the peak amplitude of input (5-4) to output (5-7) for M_2 is

$$G_{M2} = \frac{1}{2} BC_1 \tag{5-8}$$

The components after the M_2 in the adaptive loop act upon the signal terms directly, so no further transforms are required.

5.4 Stability investigation of uncompensated system and compensation

After linearizing the required components, the block diagram of the system is formed with only the fixed compcnents. All components are fixed except the smoothing filter $G_2(s)$ which is acting also as a compensator. The block diagram is reduced to a convenient form and the overall loop transfer function is obtained. The gain requirement is decided on the basis of static and dynamic accuracy and the gain is set at the lowest acceptable value. Using the preferred standard method of analysis, the uncompensated system is analyzed. The minimum gain permissible is that which provides acceptable steady state error. In most uncompensated systems this gain exceeds the critical gain of the system and the system becomes unstable. Proper compensation is necessary to make the system stable maintaining the required gain. Many different methods of compensation are available depending on situations, but whenever possible, a phase lag compensation is recommended because the compensator is also acting as a smoothing filter. The required band width is narrow in this system because the parameter drift frequency is generally very low and it is effective in suppressing noise.

5.5 Example

Some adaptive control systems are designed analytically and simulated on computer. The actual performance of the system is compared with the one predicted by analytical method. Because of difficulty in simulation on an analog computer, the first example uses mild compensation and was simulated on analog computer. In the last example an extreme case of design is made and it is simulated on digital computer.

(a) Example 1

Design an adaptive control loop to maintain optimum for the following plant in which the pole is drifting with environment change.

$$G_p(s) = \frac{81}{S(S + 9)}$$

Input to the plant; $R = 4\sin 1.884t$

Specification; Steady state error = 10% of average velocity of the parameter drift.

Maximum overshoot = less than 20%

Hunting loss = 10% of optimum parameter value

Expected maximum frequency of parameter drift = 0.1 cps.

Design steps

(1) Gain requirement

$$\text{Steady state error} = \frac{\omega_a}{K_v}$$

where; ω_a = average velocity of parameter drift

K_v = static error coefficient

In this case $K_v = 10$

(2) Selection of perturbation

Parameter perturbation signal; $C_p = C_1 \sin \omega_1 t$

Assume 1 v of parameter adjusting signal corresponds to 10% of optimum parameter value. Therefore, $C_1 = 1$ v

The expected maximum frequency of parameter drift is given as $\omega_m = 0.1$cps.

Choose $\omega_1 = 5$ cps. which gives $\omega_m : \omega_1 = 1 : 50$ and gives also no sharp overshoot in correction signal as seen in simulation study. Thus, $C_p = 1 \sin 31.4t$

(3) Establishing a linearized block diagram of the system

Referring to the Fig. 4 the complete block diagram of the system, find each block.

1) $G_m(s)$ is generally, from 3.1

$$G_m(s) = \frac{K_m}{1 + \omega_m S}$$

The measured phase characteristic of $G_m(s)$ is the same as the phase characteristic of the plant (Refer to the section 4.1(c)(2)). Therefore, the denominator of the $G_m(s)$ is the same as the plant transfer function denominator, thus

$$G_m(s) = \frac{K_m}{S(1 + 0.111S)}$$

$$\left|G_m(s)\right| = A_m = \left|\frac{K_m}{(j\,\omega_m)(1+0.111j\omega_m)}\right|$$

A_m was measured as 0.086 at $\omega_m = 0.0785$ rad/sec.

K_m is calculated as 0.122, thus

$$G_m(s) = \frac{0.122}{S(1+0.111S)}$$

2) M_1

The linearized M_1 is obtained from equation (5-3)

$$G_{M1} = 2a_1 A_p C_1$$

The a_1 is assumed to be 1. The attenuation factor of the perturbation signal A_p was measured. The value of A_p is different depending on the point of the error output signal wave at which it is measured as shown in Fig. 10(a).

$$\text{Maximum } A_p = 0.0087$$

$$\text{Minimum } A_p = 0.00199$$

$$\text{Average } A_p = 0.0054$$

Take the maximum A_p for the first trial design, $C_1 = 1$ v, thus

$$G_{M1} = 0.0174$$

The value of A_p affects the overall loop gain which will be adjusted properly at the final design.

3)

$$G_1(s) = \frac{\omega_1^2 S}{S^2 + \omega_1 S + \omega_1^2}$$

Linearized $G_{1equiv.}(s)$ is, from equation (5-5),

$$G_{1equiv}(s) = \frac{A}{1 + \tau_{1eq.} S}$$

$A = A_3$ = attenuation factor of $G_1(s)$ at ω_1, which is measured as 6.6

$$\Delta\theta = \angle G_1(j\omega_1 + j\omega_m) = 2.67°$$

$$\tan 2.67° = \tau_{1eq.} S$$

$$S = j\omega_m = j0.628$$

$$\tau_{1eq.} = \frac{\tan 2.67°}{0.628} = 0.0603$$

Thus,

$$G_{1eq.}(s) = \frac{6.6}{1 + 0.0603S}$$

4) M_2

From equation (5-8), $G_{M2} = \frac{1}{2} BC_1$

B is the attenuation factor of reference signal phase lag filter $G_3(s)$. In this example $G_3(s)$ is not used because the $G_m(s)$ gives the carrier phase shift of 160° which is closer to 180° phase shift. Therefore, B = 1

C_1 = 1 v. Thus, $G_{M2} = 0.5$

5) $G_2(s)$ will be a compensator in this sytem and is not decided yet.

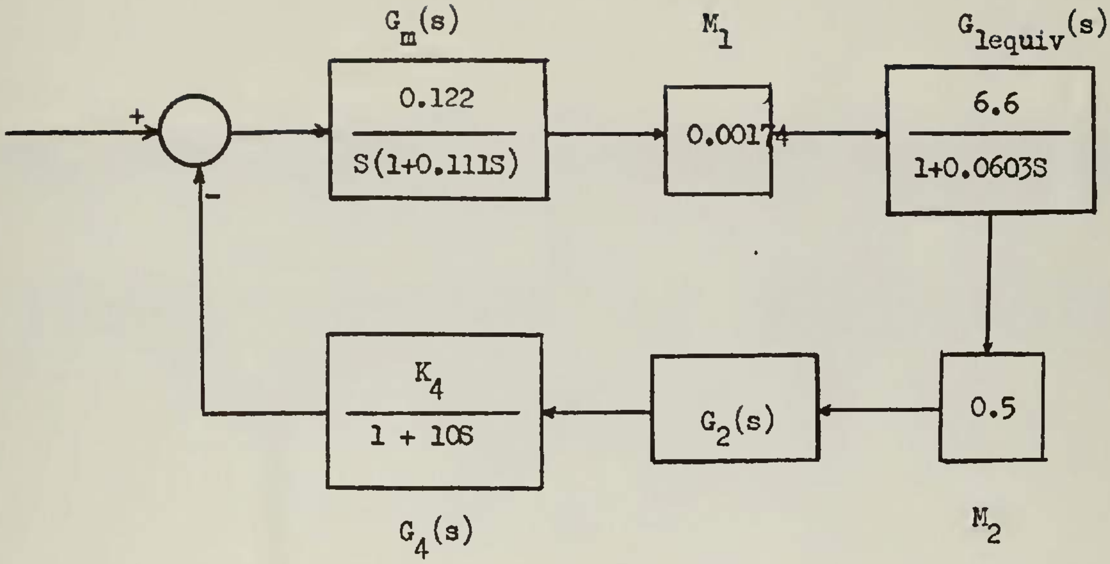
6) The integrator $G_4(s)$ is chosen to have following form because of difficulty in the simulation circuit as explained in section 4.2.

$$G_4(s) = \frac{K_4}{1 + 10S}$$

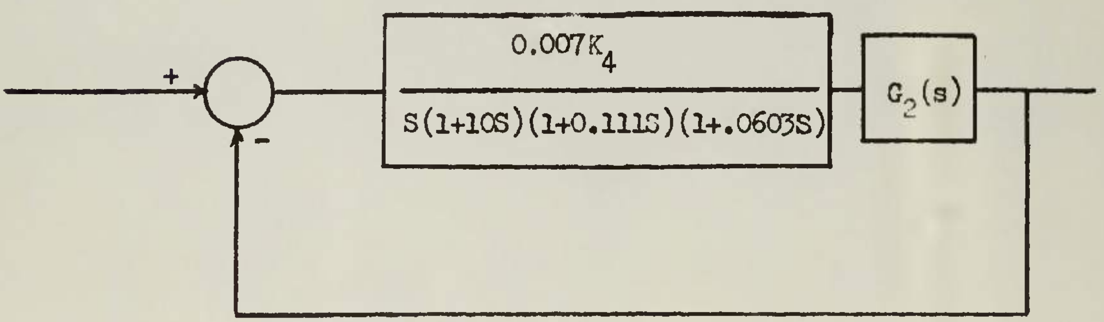
(4) The linearized block diagram of the system becomes as Fig. 35(a) and which is reduced to Figure (b). Excluding $G_2(s)$ from the system and putting $K_v = 10$, the stability of the system is investigated. To obtain $K_v = 10$, K_4 should be 1430. The open loop transfer function of uncompensated system becomes,

$$G_o(s) = \frac{10}{S(1 + 10S)(1 + 0.111S)(1 + 0.0603S)} \qquad (5\text{-}9)$$

The Bode diagram of the system is plotted as Fig. 36. The phase margin is zero and the system is unstable.



(a)



(b)

Fig. 35 The linearized block diagram of example 1.

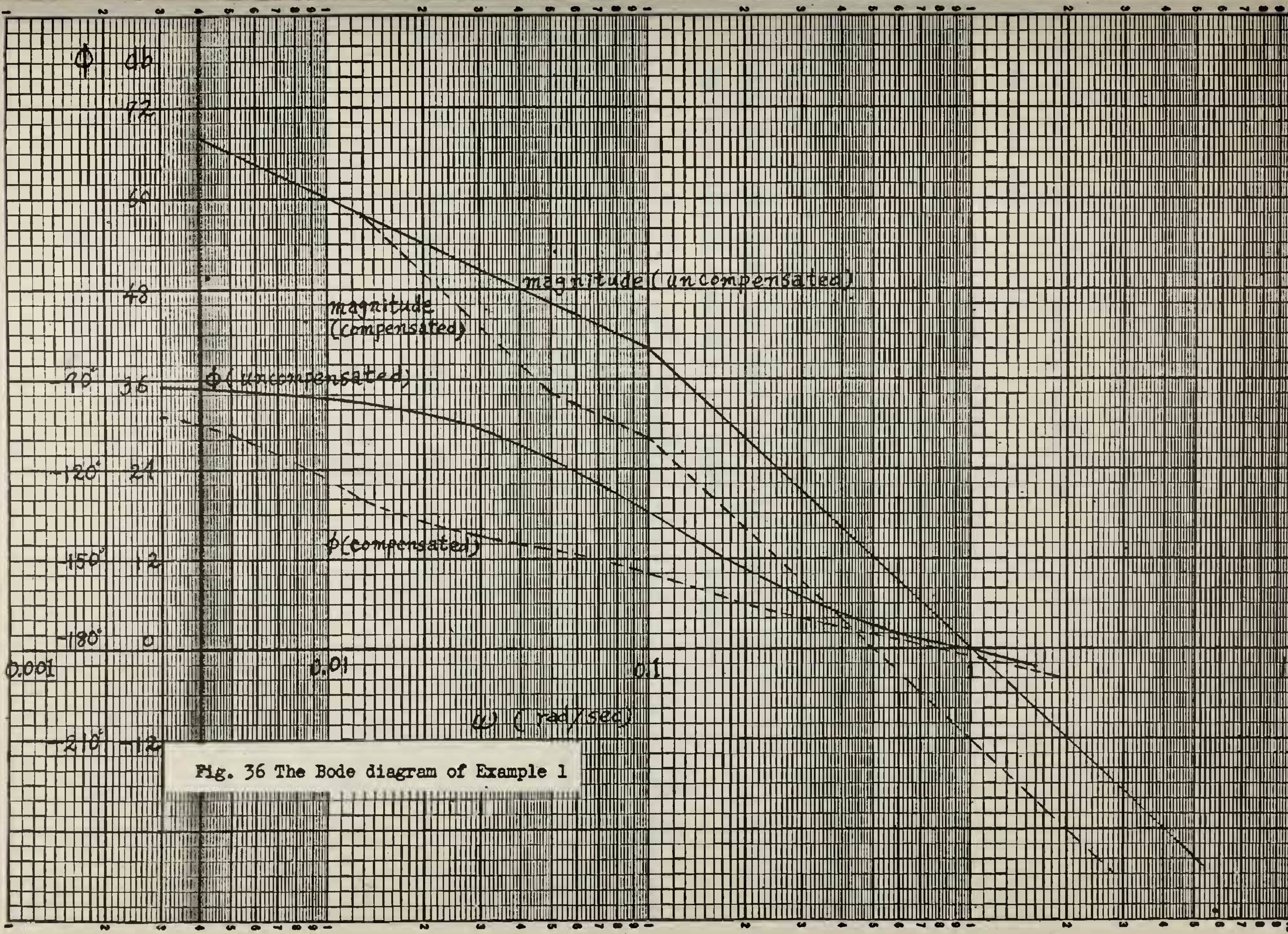


Fig. 36 The Bode diagram of Example 1

The $G_o(s)$ also can be written in standard transient response form as,

$$G_o(s) = \frac{149}{S(S + 0.1)(S + 9)(S + 16.6)} \tag{5-10}$$

The root locus is also plotted as Fig. 37.

The roots are;

$$r_1 = 0.0336 - j0.9874$$

$$r_2 = 0.0336 - j0.9874$$

$$r_3 = -9.2322$$

$$r_4 = -16.5349 \tag{5-11}$$

Two roots are located in the right half of the S plane. Thus the root locus plot also shows the instability of the system.

The characteristic equation of the closed loop system is,

$$S^4 + 25.7S^3 + 151.9S^2 + 14.9S + K = 0 \tag{5-11}$$

By Routh's criterion, the critical gain of the system is found as $K_c = 87.9$

The required gain of the system is 149 and exceeds the critical gain.

A compensation is necessary to make the system stable.

(5) Compensation

A lag compensation is adopted, the compensator is

$$G_2(s) = \frac{1 + aTS}{1 + TS} \tag{5-12}$$

Compensation 1;

To get 5° phase margin in the final compensated system, a cross-over frequency $\omega_c = 0.5$ (phase margin is 7.5° on Bode diagram) is chosen in Fig. 36.

aT is calculated from the following equation,

$$\frac{1}{aT} = \frac{\omega_c}{10} \tag{5-13}$$

$$\frac{1}{aT} = \frac{0.5}{10} \qquad aT = 20 \tag{5-14}$$

$$\left| G_o(j\omega_c) \right| = 12 \text{ db on Bode diagram Fig. 36}$$

This must be attenuated, therefore,

$$20 \log a = -12 \qquad\qquad a = \frac{1}{3.98}$$

Substitute this value in equation (5-14) and get T = 79.6. Thus,

$$G_2(s) = \frac{1 + 20\ S}{1 + 79.6\ S} \tag{5-15}$$

This is the required compensator to have 5° phase margin.

(6) Investigation of stability of the compensated system.

Putting this $G_2(s)$ into the original system and you get the open loop transfer function of the compensated system as,

$$G_{oc}(s) = \frac{10(1+20S)}{S(1+79.6S)(1+10S)(1+0.111S)(1+0.0603S)} \tag{5-16}$$

The Bode diagram of the compensated system is plotted in Fig. 36. Now, the system has 5° phase margin and expected to be stable.

The open loop transfer function of the compensated system is rewritten in standard transient response form as,

$$G_{oc}(s) = \frac{37.45(S+0.05)}{S(S+0.0126)(S+0.1)(S+9)(S+16.6)} \tag{5-17}$$

The characteristic equation of the closed loop system is

$$S^5+25.713S^4+152.224S^3+16.813S^2+37.638S+1.873 = 0 \tag{5-18}$$

The roots are

$$r_1 = -0.009526 + j0.4974$$

$$r_2 = -0.009526 - j0.4974$$

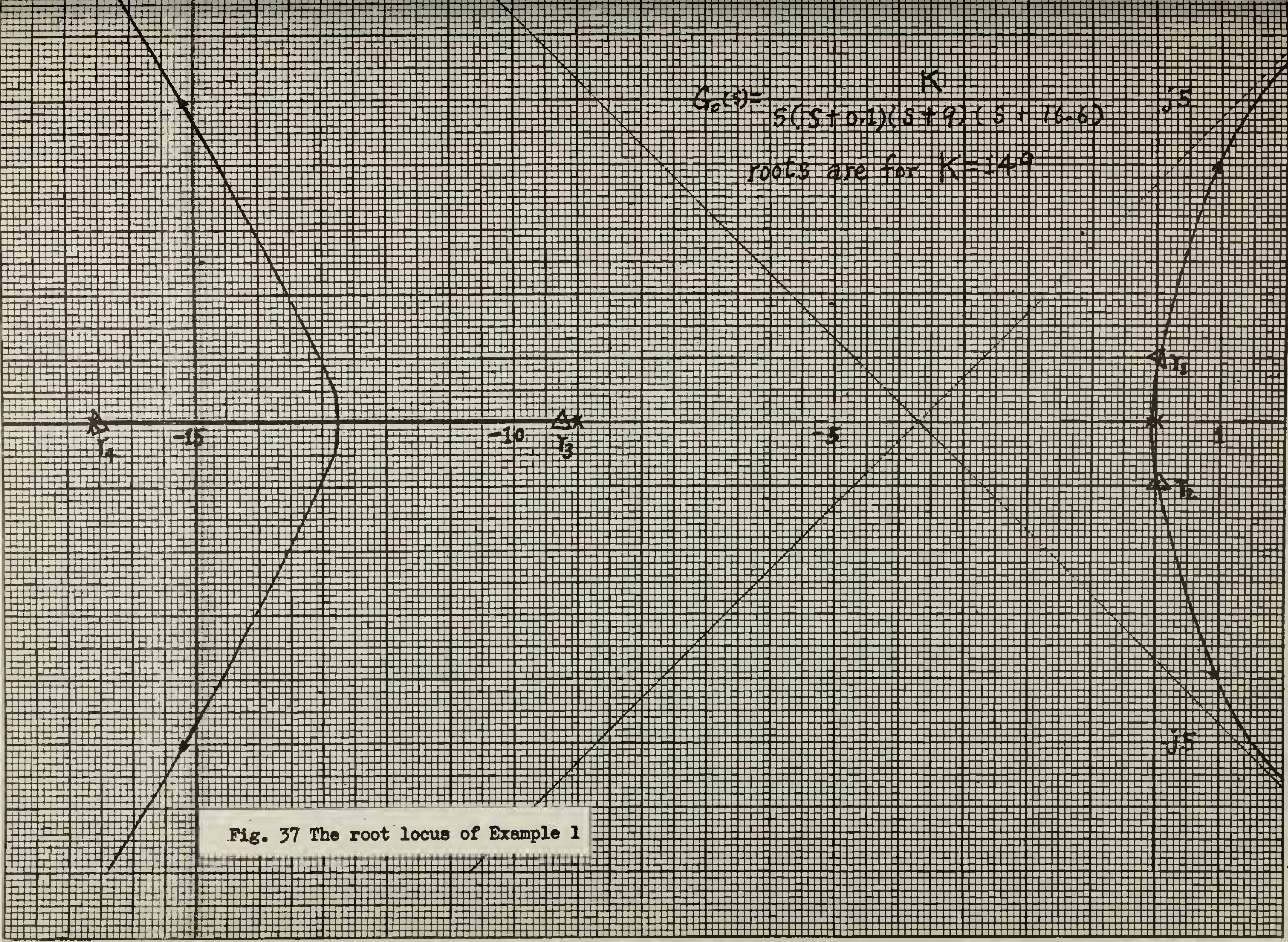


Fig. 37 The root locus of Example 1

$r_3 = -0.05039$

$r_4 = -9.0531$

$r_5 = -16.5905$

All roots are in the left half of the S plane.

Approximated ζ of the compensated system is found as 0.01916

M_{pt} for this $\zeta = 1.9417$

(7) Simulation of the compensated system on analog computer

The compensated system is simulated on analog computer. The actual response of the system is recorded as Fig. 38 where (a) is the parameter drift being corrected and (b) is the corresponding correction signal. The response of the system is different depending on the direction of drift of parameter as explained in Section 4.1(b). We take the worst case of response, that is, the response of the system when the parameter drifts to the direction of decreasing system ζ . From the recorded result of the system response, following data are obtained.

Steady state error = 16.5%

M_{pt} = approximately 2

(8) Comparison of theoretical and actual data of response

	theoretical	actual
steady state error	10%	16.5%
M_{pt}	1.9417	2.0

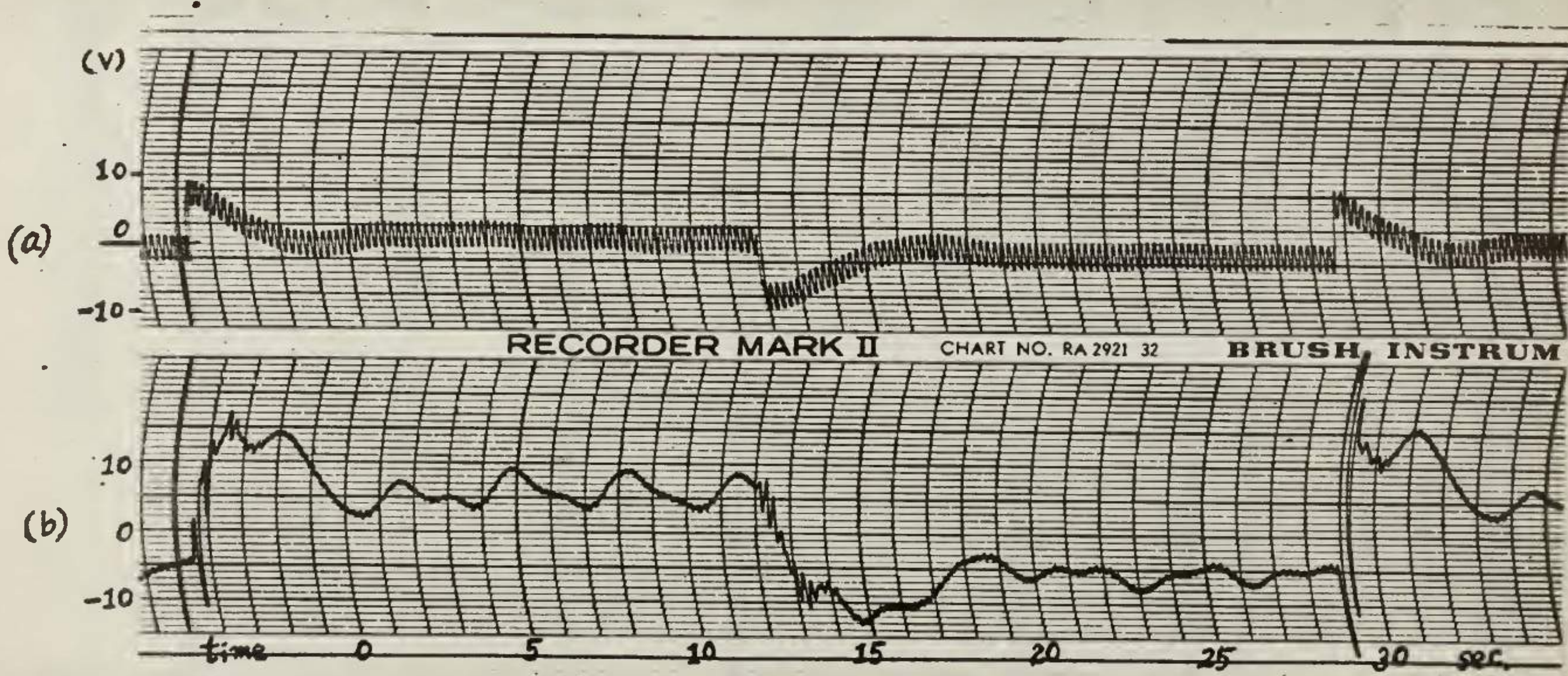


Fig. 38 The response of the system, Example 1.

(a) Parameter drift being corrected

(b) Corresponding correction signal

(b) Example 1.1

A different compensator is designed for the example 1 and the compensated system is simulated on analog computer. The response of the actual system is also compared with the response predicted analytically.

Compensation 2 ;

(1) To get 10° phase margin in the final compensated system, $\omega_c = 0.3$ (phase margin is 17° on Bode diagram) is chosen from uncompensated system Bode diagram.

Using equation (5-13), $aT = 33.3$

$|G_o(j\omega_c)| = 20.5$ db (From Bode diagram of uncompensated system)

$20 \log a = -20.5 \qquad a = \dfrac{1}{10.6}$

$T = 353$

Thus,
$$G_2(s) = \frac{1 + 33.3S}{1 + 353\,S} \tag{5-19}$$

This is the required compensator to have 10° phase margin in the final compensated system.

(2) Investigation of stability of the compensated system

Putting the above $G_2(s)$ into the original system and get the open loop transfer function of the compensated system as,

$$G_{oc}(s) = \frac{10(1+33.3S)}{S(1+353S)(1+10S)(1+0.111S)(1+0.0603S)} \tag{5-20}$$

The Bode diagram of the compensated system is plotted in Fig. 39. Now, the system has 10° phase margin.

The open loop transfer function is rewritten in standard transient response form as,

$$G_{oc}(s) = \frac{14.1\,(S+0.03)}{S(S+0.00284)(S+0.1)(S+9)(S+16.6)} \tag{5-21}$$

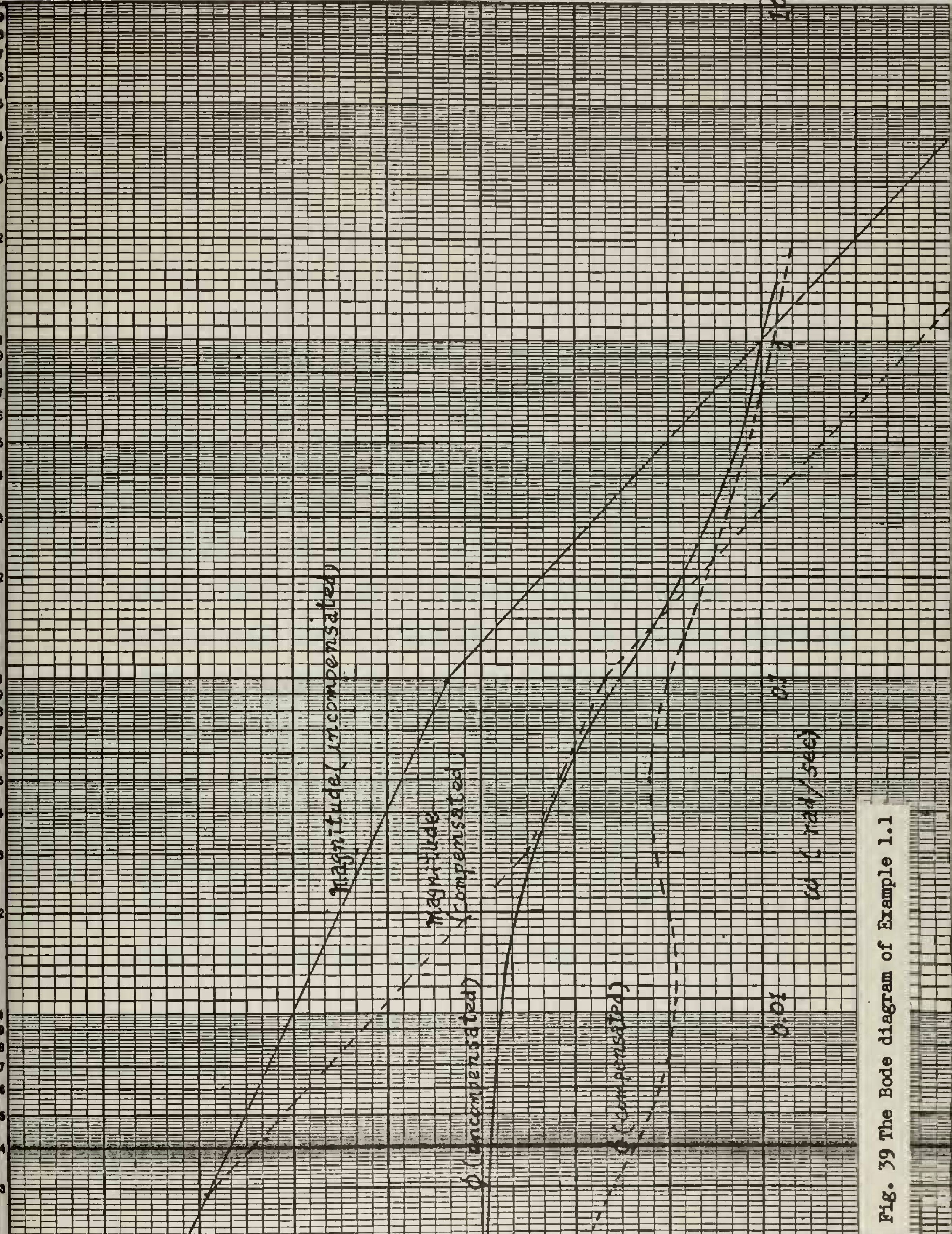


Fig. 39 The Bode diagram of Example 1.1

The characteristic equation of the closed system is

$$S^5+25.70284S^4+151.973S^3+15.328S^2+14.1423S+0.423 = 0 \qquad (5\text{-}22)$$

The roots are

$r_1 = -0.02784 + j0.3025$

$r_2 = -0.02784 - j0.3025$

$r_3 = -0.03062$

$r_4 = -9.0157$

$r_5 = -16.6008$

All roots are in the left half of the S plane.

Approximate ζ of the compensated system is 0.0915

M_{pt} for this ζ = 1.758

(3) Simulation of the compensated system on analog computer

The compensated system is simulated on analog computer. The result is shown in Fig. 40, where (a) is the parameter drift being corrected, (b) is the corresponding correction signal, (c) is the demodulated output and (d) is the error between the plant output and model output.

From the results:

Steady state error = 11.1%

$M_{pt} = 2$

(4) Comparison of theoretical and actual data of response

	theoretical	actual
steady state error	10%	12.1%
M_{pt}	1.758	2.0

(5) Simulation of compensated system on digital computer

Because of difficulty in the simulation of the required compensator on analog computer, an approximate simulation of the required compensator

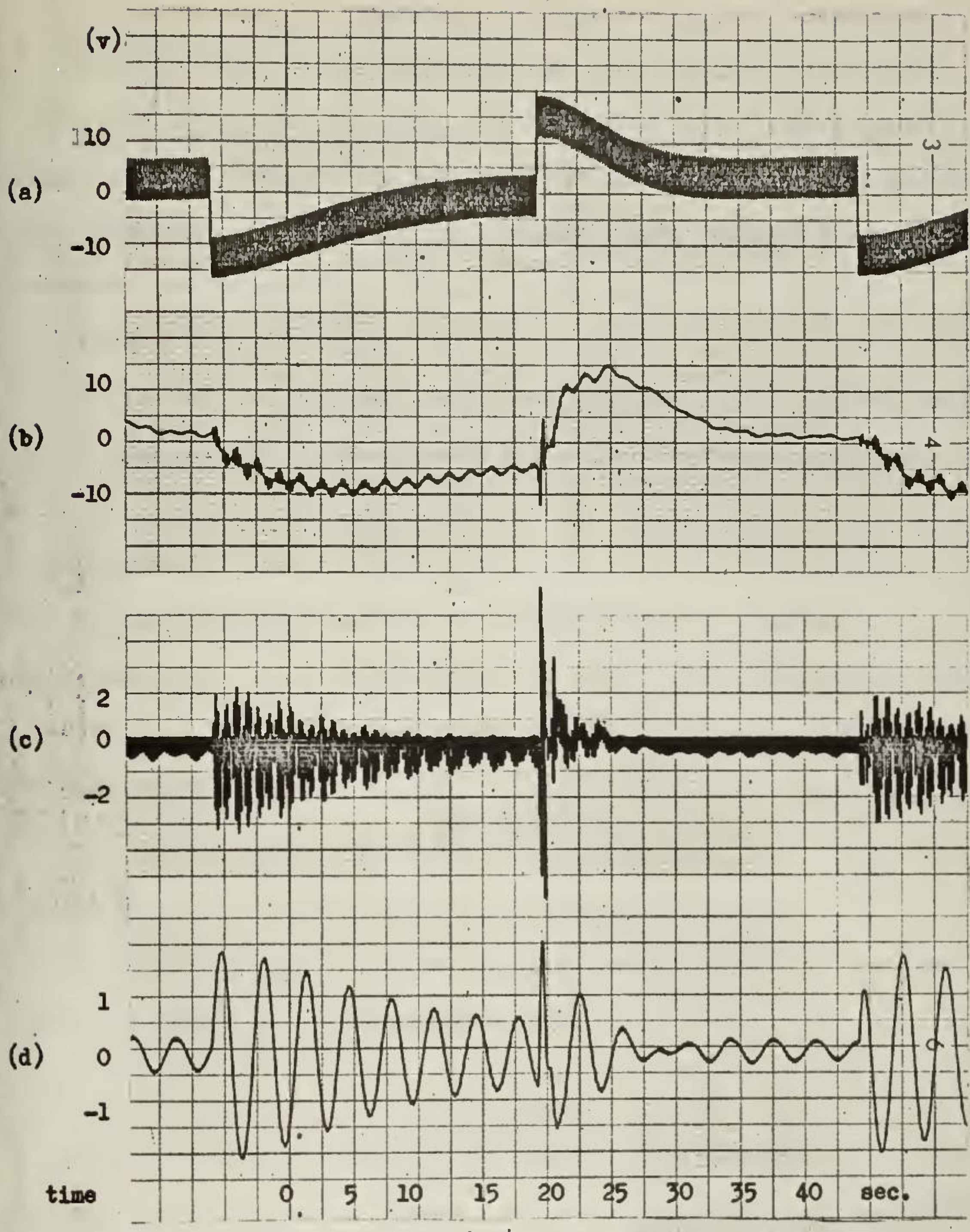


Fig.40 Example 1.1 response of compensated system
(a) parameter drift being corrected
(b) Correction signal
(c) Demodulated output
(d) Error

was used in the simulation on analog computer in (3).

The same system is simulated on a digital computer and the result is compared. Refer to the Appendix I for the FORTRAN program of the simulation. Fig. 41 is the graph of correction signal plotted from the numerical data output of the computer. If the Fig. 41 is compared with Fig. 40(b) there is a difference in signal shape. This is due to the approximate simulation of the compensator in Fig. 40.

(c) Example 1.2

To get more phase margin in the system of Example 1, another compensator is designed and the compensated system is simulated on a digital computer.

Compensation 3:

(1) To get 40° phase margin in the final compensated system, $\omega_c = 0.1$ (phase margin is 45° on Bode diagram) is chosen from uncompensated system Bode diagram. Calculated in the same way as in example 1, the required compensator is found as

$$G_2(s) = \frac{1 + 100S}{1 + 10^4 S}$$

(2) Investigation of stability of the compensated system

Putting this $G_2(s)$ into the original system and get the open loop transfer function of the compensated system as,

$$G_{oc}(s) = \frac{10(1+100S)}{S(1+10^4 S)(1+10S)(1+0.111S)(1+0.0603)}$$

The Bode diagram of the compensated system is plotted in Fig. 42. Now, the system has 40° phase margin.

The open loop transfer function is rewritten in standard transient response form as

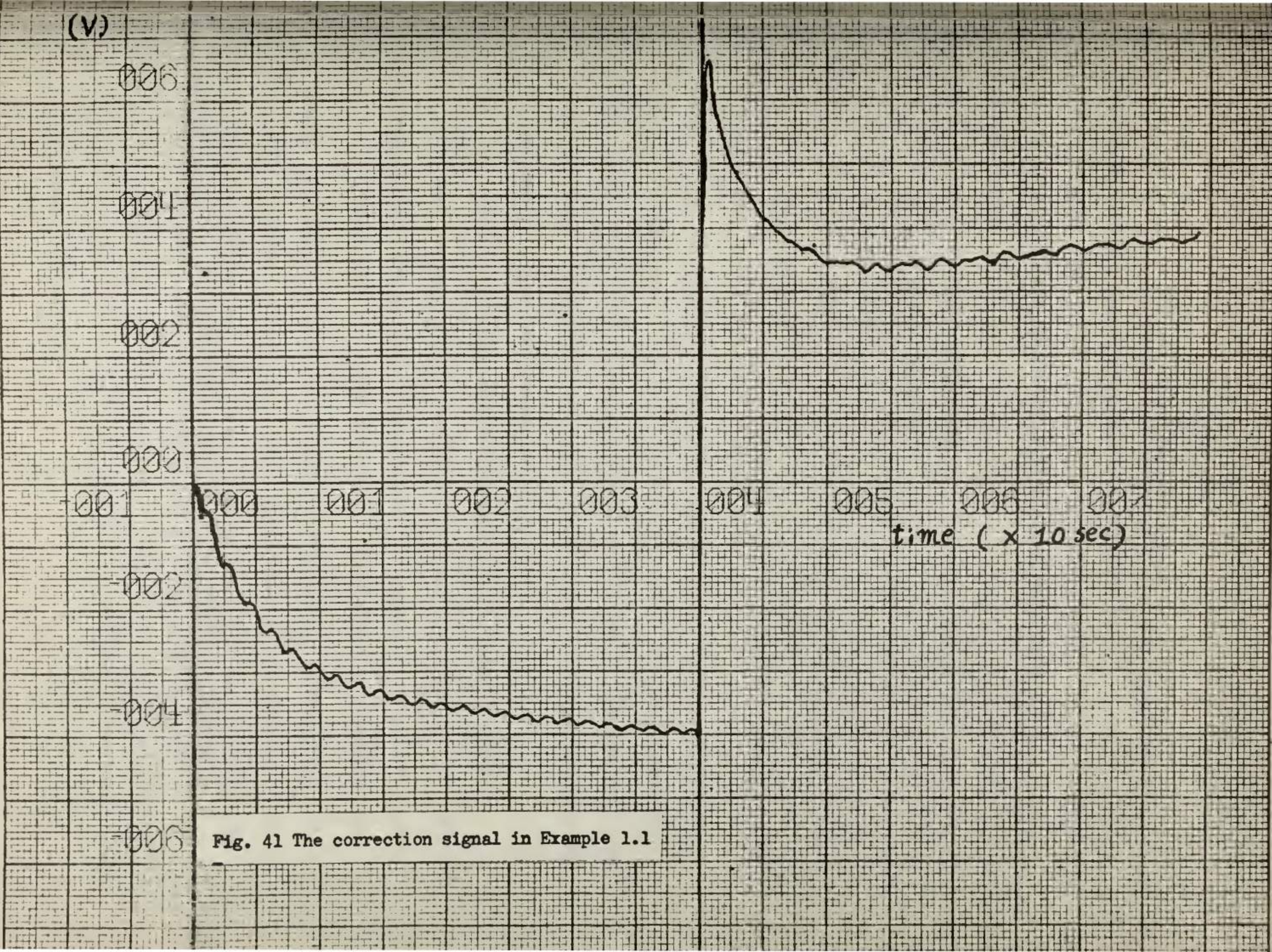


Fig. 41 The correction signal in Example 1.1

$$G_{oc}(s) = \frac{1.49(S+0.01)}{S(S+10^{-4})(S+0.1)(S+9)(S+16.6)}$$

The characteristic equation of the closed system is

$$S^5+25.7001S^4+151.90257S^3+14.91519S^2+1.49149S+0.0149 = 0$$

The roots are

$$r_1 = -0.04353 + j0.08431$$

$$r_2 = -0.04353 -j0.08431$$

$$r_3 = -0.01108$$

$$r_4 = -8.9951$$

$$r_5 = -16.6069$$

All roots are in the left half of the S plane.

From the pair of complex roots, ζ of the system is found as 0.459 and the M_{pt} for this value of ζ is 1.195. But this is no more true because these complex roots are no more dominant in the system. The dominant root is r_3 and real. Accordingly the expected response of the system should be an overdamped one and the time constant is large.

(3) The simulation of the compensated system on a digital computer

The compensated system is simulated on a digital computer. The FORTRAN programming for the simulation on a digital computer is shown in Appendix I.

In calculating K_4 for the simulation, considering an overdamped response, $A_p = 0.00199$ which is the minimum value of A_p is used and the K_4 is found as 6240 to get $K_v = 10$. The result of the simulation is shown in Fig. 43 which shows the pole drift being corrected and the corresponding correction signal. The Fig. 44 shows the pole drift being corrected and the error. From the results, the steady state error can not be found exactly because the signal does not reach the steady state in 40 seconds used in the computer simulation.

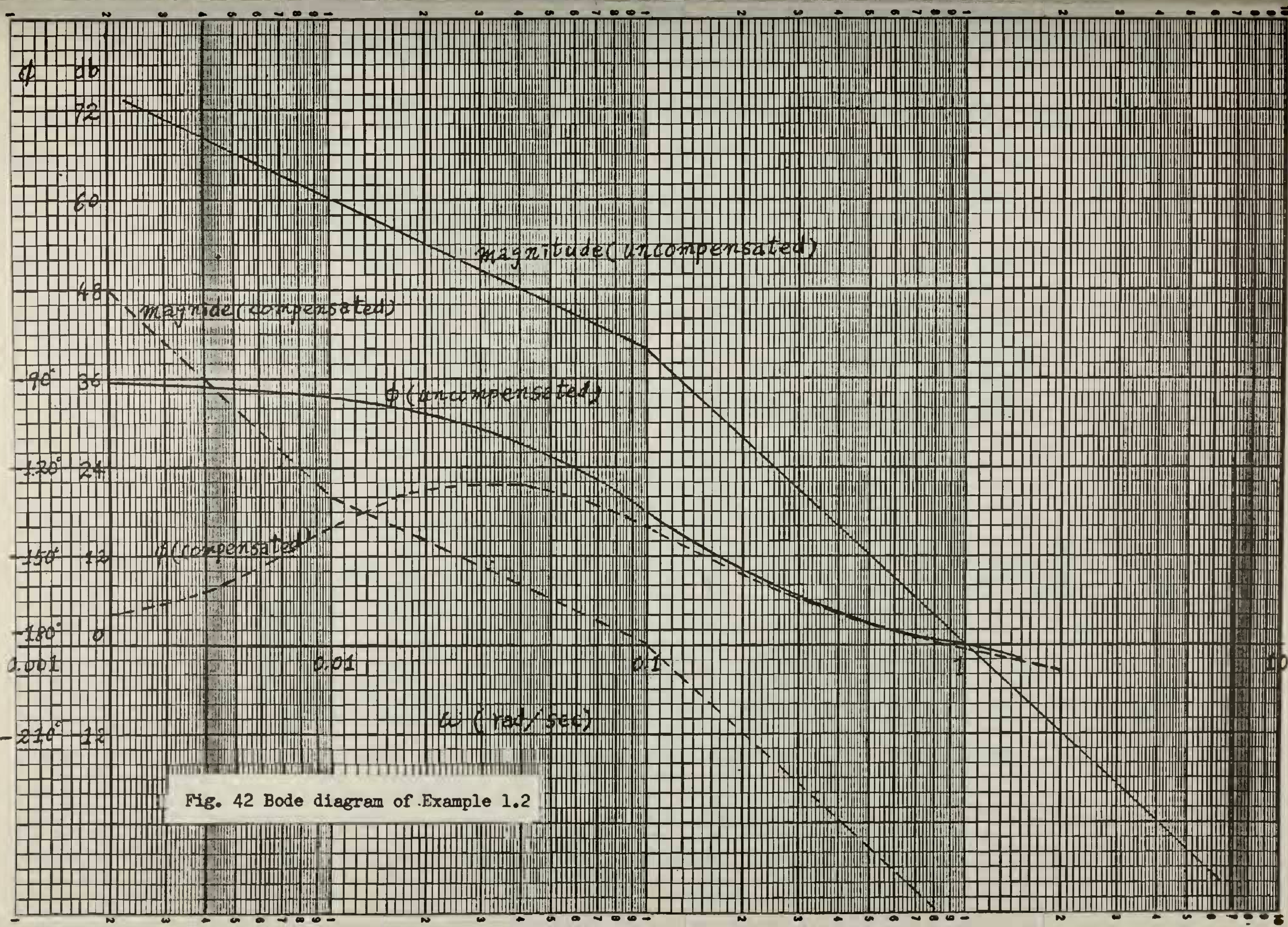


Fig. 42 Bode diagram of Example 1.2

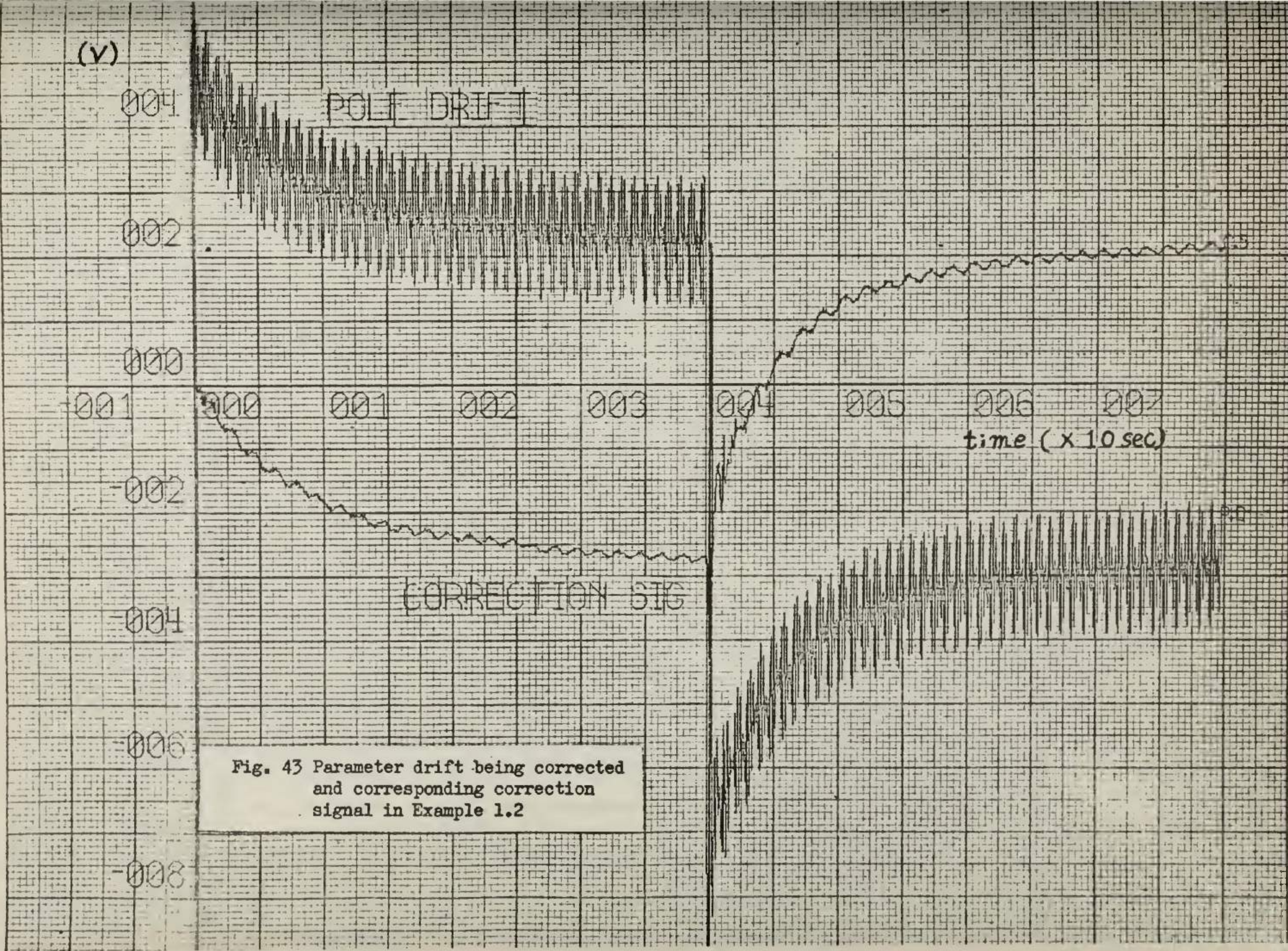


Fig. 43 Parameter drift being corrected and corresponding correction signal in Example 1.2

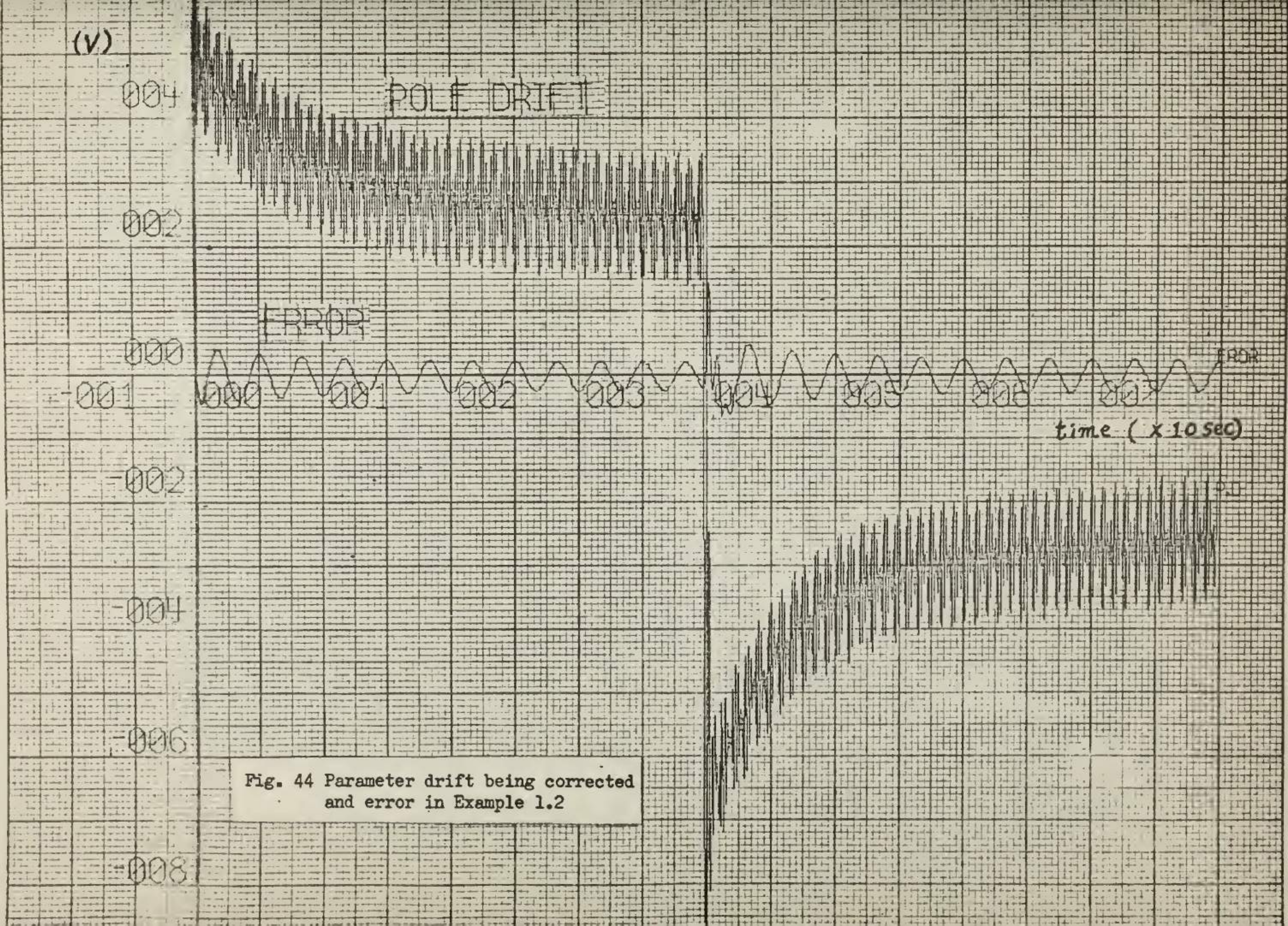


Fig. 44 Parameter drift being corrected and error in Example 1.2

(4) Comparison of theoretical and actual response

It is predicted theoretically that the compensated system response is overdamped and the time constant is large. The actual response shows the same pattern of response as predicted. The theoretical steady state error is 10%. The actual steady state error could not be measured but is expected to reach close to the theoretical one.

(d) Example 2

To test the validity of the analytical method in an extreme case of design, the steady state error in example 1 is changed to 1% and rigorous compensation is used. All other specifications are the same as example 1.

(1) The required $K_v = 100$

(2) The open loop transfer function of uncompensated system is

$$G_o(s) = \frac{100}{S(1 + 10S)(1 + 0.111S)(1 + 0.0603S)} \tag{5-23}$$

The Bode diagram of uncompensated system is plotted as Fig. 45. The phase margin is -29° and the system is unstable.

The $G_o(s)$ is also rewritten in standard transient response form as,

$$G_o(s) = \frac{1490}{S(S + 0.1)(S + 9)(S + 16.6)} \tag{5-24}$$

The characteristic equation of closed loop system is

$$S^4 + 25.7S^3 + 151.9S^2 + 14.9S + 1490 = 0 \tag{5-25}$$

The roots are

$r_1 = 0.6049 + j\ 2.8461$

$r_2 = 0.6049 - j\ 2.8461$

$r_3 = -11.209$

$r_4 = -15.7008$

By Routh's criterion, the critical gain is found as $K_c = 87.9$.

The required gain 1490 exceeds the critical gain.

(3) Compensation

A lag compensation is adopted. To get 40° phase margin in the final compensated system, cross-over frequency $\omega_c = 0.1$ (phase margin is 45° on Bode diagram) is chosen from uncompensated system Bode diagram, Fig. 45. Calculated in the same way as in example one the required compensator is found as,

$$G_2(s) = \frac{1 + 100S}{1 + 10^5 S} \tag{5-26}$$

(4) Investigation of stability of the compensated system

Putting this $G_2(s)$ into the original system and get the open loop transfer function of the compensated system as,

$$G_{oc}(s) = \frac{100(1+100S)}{S(1+10^5 S)(1+10S)(1+0.111S)(1+0.0603S)} \tag{5-27}$$

The Bode diagram of the compensated system is plotted in Fig. 45. Now, the phase margin is 39°. The open loop transfer function is rewritten in standard transient response form as,

$$G_{oc}(s) = \frac{1.49(S+0.01)}{S(S+0.00001)(S+0.1)(S+9)(S+16.6)} \tag{5-28}$$

The characteristic equation of closed system is

$$S^5 + 25.70001S^4 + 151.900257S^3 + 14.90151S^2 + 1.490149S + 0.0149 = 0 \tag{5-29}$$

The roots are

$r_1 = -0.04348 + j\,0.08428$

$r_2 = -0.04348 - j0.08428$

$r_3 = -0.01109$

$r_4 = -8.995$

$r_5 = -16.6069$

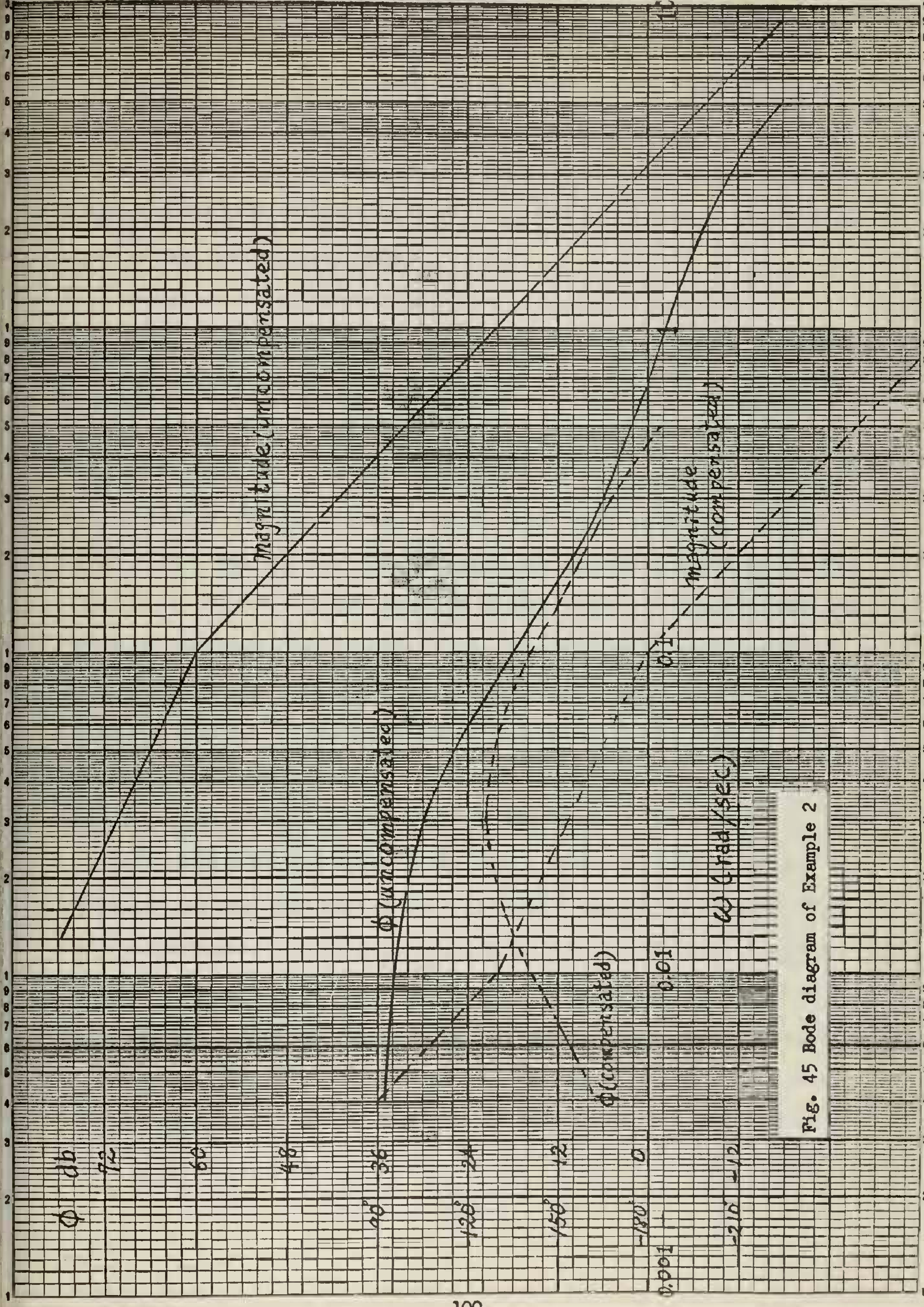


Fig. 45 Bode diagram of Example 2

All roots are in the left half of the S plane and are the same values as the roots in the Example 1.2. Thus the same response as in the example 1.2 is expected, that is, the response of the compensated system is overdamped and the time constant is large. Furthermore, a smaller steady state error is expected theoretically.

(5) Simulation of the compensated system on a digital computer

The compensated system is simulated on a digital computer. The FORTRAN programming for the simulation on a digital computer is shown in Appendix I. In calculating K_4 for the simulation, $A_p = 0.00199$ is used and the K_4 is found as 62400 to get $K_v = 100$. Considering the overdamped response, K_4 is increased by 35% of the calculated value to 84000 in order to see the effect of an increased gain. The system still maintains stability and the rise time is reduced a little compared to the response of example 1.2. The result of the simulation is shown in Fig. 46 which shows the pole drift being corrected and the corresponding correction signal. From the result, the steady state error can not be found because the signal does not reach the steady state yet.

(6) Comparison of theoretical and actual response

It is predicted theoretically that the compensated system response is overdamped and the time constant is very large. The actual response shows the same pattern as predicted and also shows that the 35% more loop gain than the designed loop gain does not hamper the stability of the system at all, which tells that the system is in very stable condition as designed. The theoretical steady state error is 1%. The actual steady state error could not be measured but the deviation of the steady state error is expected to be not so large.

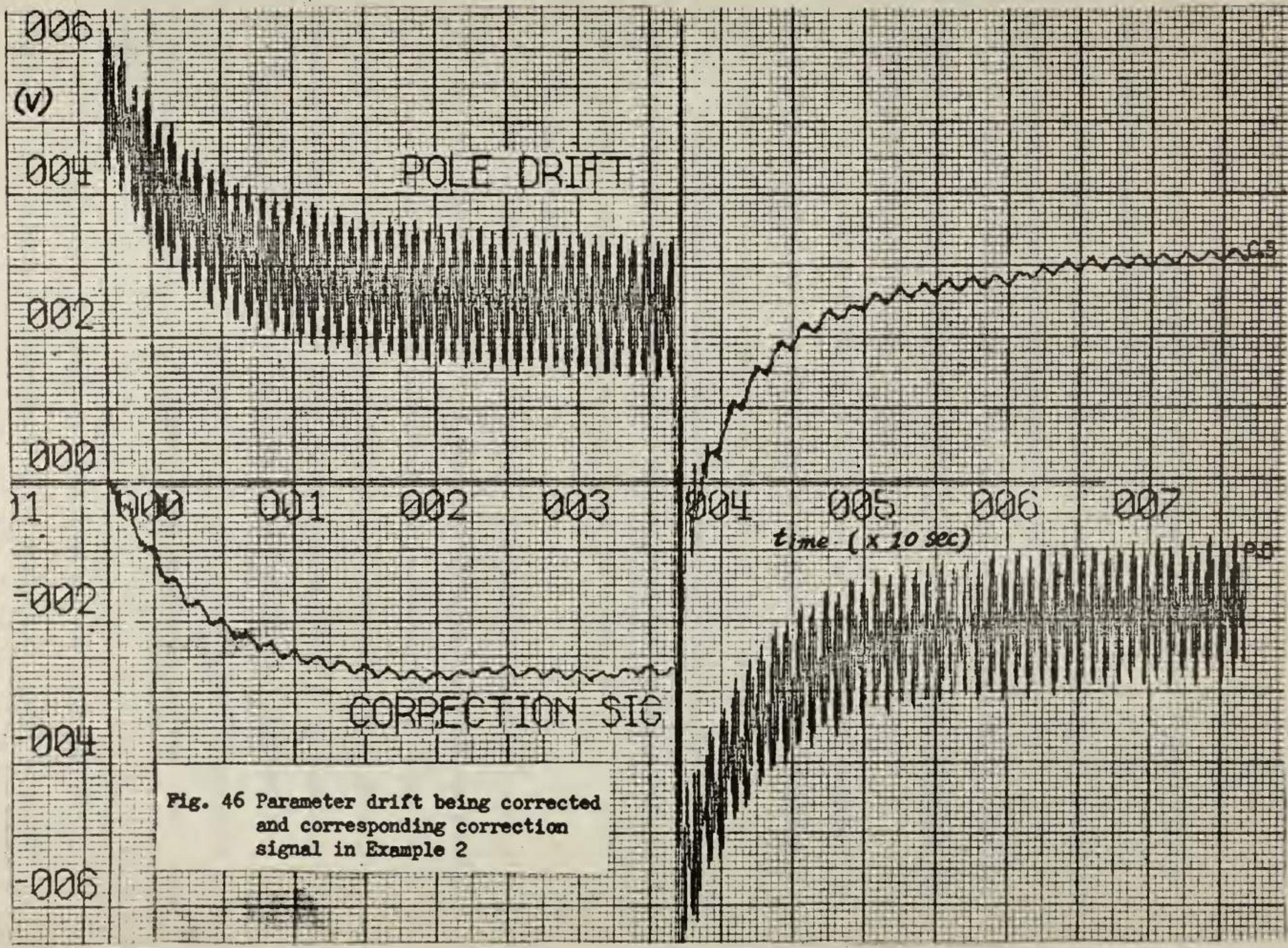


Fig. 46 Parameter drift being corrected and corresponding correction signal in Example 2

6. Conclusions.

(a) Analysis of the system.

The open loop signal analysis showed clearly the feasibility of the adaptive scheme in this system.

The signal analysis leads to the obtaining of a correct equivalent signal transfer block for the component in the system.

The simulation study of the successive adaptive circuit, from the simplest to the complete circuit, showed the function of each component and the effect of component change in the circuit.

It is confirmed that only the parameter drift signal component is acting through the closed adaptive loop; accordingly only the parameter drift signal need to be considered in the closed loop analysis.

The simulation study showed that the analytical explanation of the system operations such as modulation, demodulation and filtering, are in close agreement with the actual operation of the system.

(b) Design of the system.

The obtaining of a correct error measurement transfer function $G_m(s)$ and the attenuation factor of perturbation signal through the plant (A_p) is most important in the analytical design because these affect the accuracy of the analytical design the greatest. The value of A_p differs depending on the point of the error output wave at which it is measured. When the compensated system is expected to be underdamped, high value of A_p is used in the simulation and when the compensated system is expected to be overdamped, low value of A_p is used in the simulation for the first trial design.

All the components are properly linearized and an analytical design is attempted. The predicted performance of the analytical design is in close

agreement with the actual system performance as verified in examples.

In the case of a compensated system which has a pair of dominant complex roots (Example 1 and 1.1) the steady state error deviation is within 10% and M_{pt} deviation is within 15% between the predicted and actual value.

In the case of a compensated system in which the pair of complex roots are not dominant and a real root is dominant, the comparison could not be made numerically, but the actual time performance had the same pattern as predicted.

This analytical method of design simplifies the design considerably and is useful in obtaining a preliminary design which will be refined by an experimental verification.

It is further suggested that this linearized technique is applied to an analysis and design of multi-loop sinusoidal perturbation adaptive control system with some modifications for the intercoupling of adaptive loops, which should be further investigated.

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APPENDIX I

FORTRAN PROGRAMMING FOR THE SIMULATION OF A SINUSOIDAL PARAMETER PERTURBATION ADAPTIVE CONTROL SYSTEM

To program the simulation of the system, the original block diagram of the system is rearranged to a proper form for programming as shown in Fig. 47 which is the rearranged block diagram of the complete adaptive control circuit, Fig. 32. Then, a simultaneous differential equations are set up in the form shown at the end of the Program Adapt 1 (page 109). The Program Adapt 1 in the following two pages, is the FORTRAN program for the simulation of the compensated system in Example 1.1. The input data and the graph control data are shown in proper form in page 110 . A sample numerical output is shown on page 111.

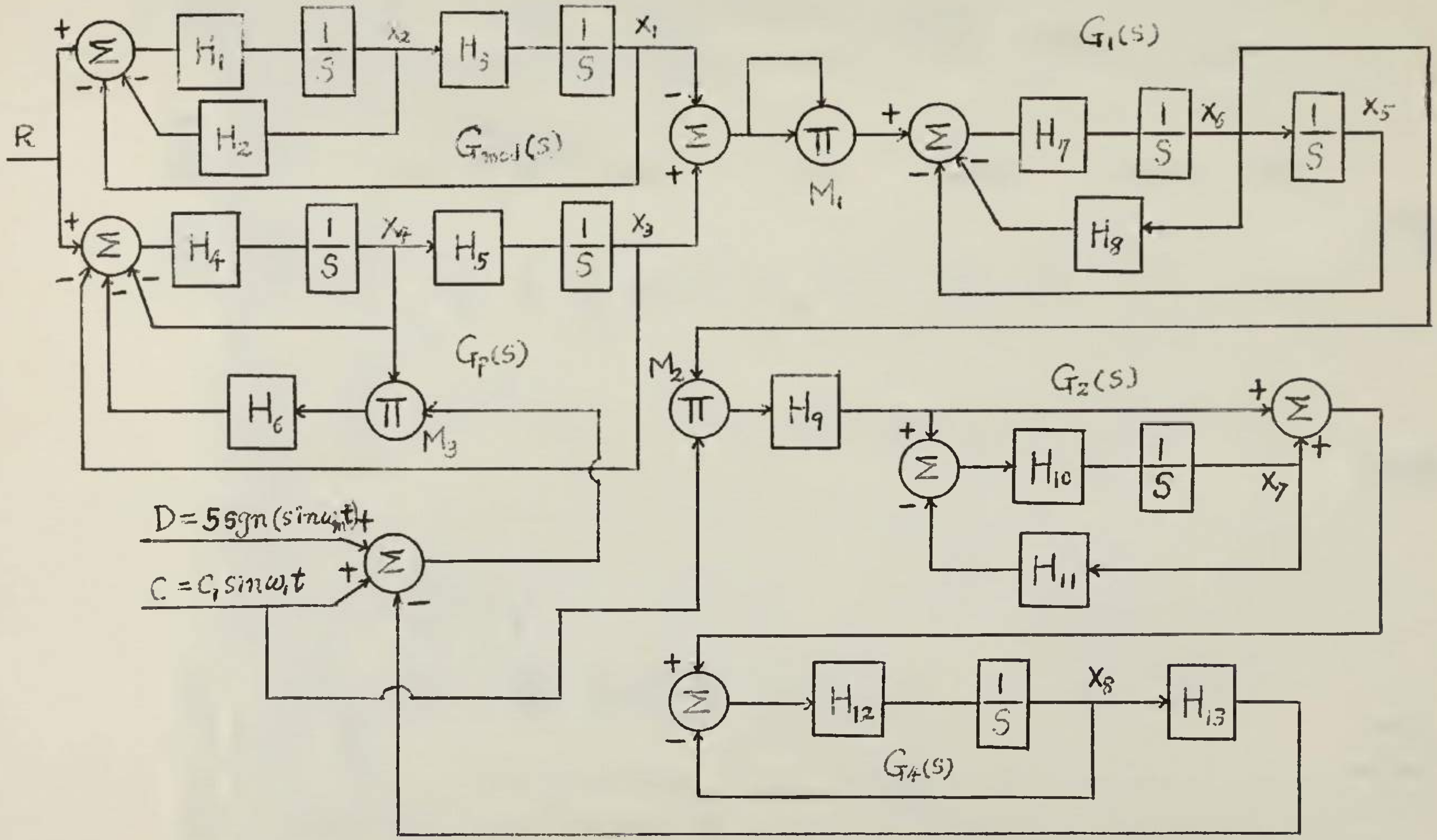

Fig. 47 Rearranged block diagram of a sinusoidal perturbation adaptive control system for the simulation on a digital computer.

```
      PROGRAM ADAPT 1
C N=NO.OF EQNS., NP=NO. OF INCREMENTS BETWEEN PRINT OUTS, TO=INITIAL
C TIME, TF=FINAL TIME, DT=TIME INCREMENT, TN=TIME FIRST PRINTED DATA,
C X(J)=INITIAL VALUES OF THE VARIABLES X(J)
C NG=NO. OF INCREMENTS BETWEEN GRAPH POINTS
C  CORSIG=CORRECTION SIGNAL    POLSIG=POLEDRIFT SIGNAL
C  C=PERTURBATION       D=POLE DRIFT            R=PLANT INPUT
     0DIMENSION X(30),X1(900),X2(900),X3(900),X4(900),X5(900),X6(900),
     1X7(900),X8(900),TIME(900),H(30)
      COMMON C,D,R,T2,H
  100 FORMAT (3I10/(8F10.4))
  102 FORMAT ( I2 / (I2,F20.0))
  200 FORMAT (F8.3,10E11.5)
  2010FORMAT(  57H   TIME    MODEL     PLANT     ERROR     CORSIG    POL
     1SIG      //)
  202 FORMAT (/)
  203 FORMAT (1H1)
  204 FORMAT (//)
  210 FORMAT (20H COEFFICIENT NUMBER I2,  2H =  F12.4, 1H.  )
      READ 100, N, NP, NG, TO, TF, DT, TN, (X(J), J=1,N)
      READ 102, NH, (  K, (H(K)), I = 1,NH )
      DO 6  I = 1,NH
      PRINT 202
    6 PRINT 210, I, H(I)
      PRINT 203
      PRINT 100, N, NP, NG, TO, TF, DT, TN, (X(J),J=1,N)
      PRINT 203
      PRINT 201
      T=TO
      T2 = 0.0
      NUMPTS = 0
      MPTS= 0
    1 ERROR=X(3)-X(1)
      CORSIG=H(13)*X(8)
      POLSIG =C+D+CORSIG
      IF (T-TN) 36, 2, 2
    2 IF(MPTS)  30,35,30
   30 IF(XMODF(MPTS,50*NP)) 31,33,31
   31 IF(XMODF(MPTS,10*NP)) 32,34,32
   32 IF(XMODF(MPTS,    NP)) 36,35,36
   33 PRINT 203
      PRINT 201
   34 PRINT 202
   35 PRINT 200, T, X(1),X(3),ERROR,CORSIG,POLSIG
   36 MPTS=MPTS+1
   37 IF(XMODF(MPTS, NG)) 38, 39, 38
   39 NUMPTS = NUMPTS+1
      IF (900 - NUMPTS ) 38, 38, 40
   40 X1(NUMPTS) = ERROR
      X2(NUMPTS) = POLSIG
      X3(NUMPTS) = CORSIG
      X4(NUMPTS) = POLSIG
      TIME (NUMPTS) = T
```

```
38 IF(TF-T-DT)  10,20,20
10 PRINT 204                                                             00480
   CALL GRAPH ( NUMPTS , TIME , X1 , 8 )
   CALL GRAPH (NUMPTS , TIME , X2 , 8 )
   CALL GRAPH ( NUMPTS , TIME , X3 , 8 )
   CALL GRAPH ( NUMPTS , TIME , X4 , 8 )
   STOP                                                                  00510
20 CALL RKUTTA(N, T, X, DT)                                              00520
   T2= T2 + DT
   T = T + DT                                                            00530
   GO TO 1                                                               00540
   END                                                                   00550
   SUBROUTINE RKUTTA (N,T,X,DT)
   DIMENSION  X(30),AK(4,30) , XDOT(30), XC(30),C(4) , H(30)
   COMMON C,D,R,T2,H
   C(1) =0.0
   C(2) =0.5
   C(3)=0.5
   C(4)=1.0
   DO 4  I=1,4
   TC=T + C(I)* DT
   DO 2 J=1,N
 2 XC(J)= X(J) +C(I)* AK(I-1,J )
   CALL DERIV(TC,XC,XDOT )
   DO 4  J=1,N
 4 AK(I,J)= DT* XDOT(J)
   DO 3 J=1,N
 3 X(J)=X(J)+(AK(1,J)+2.*AK(2,J)+2.* AK(3,J)+AK(4,J))/ 6.
   RETURN
   END
   SUBROUTINE DERIV(T, X, XDOT)
   DIMENSION  XDOT(30), X(30), H(30)
   COMMON C,D,R,T2,H
   IF(40.-T2) 3,3,1
 1 D=5.0
   GO TO 2
 3 D=-5.0
   IF(80.-T2) 4,4,2
 4 T2=0.0
   GO TO 1
 2 C=1.0*SINF(31.4*T)
   R=4.*SINF(1.884*T)
   XDOT(1)=H(3)*X(2)
   XDOT(2)=H(1)*(R-X(1)-X(2))
   XDOT(3)=H(5)*X(4)
   XDOT(4)=H(4)*(R-X(3)-X( 4)-H(6)*X(4)*(C+D+H( 13)*X(8)))
   XDOT(5)=X(6)
   XDOT(6)=H(7)*((X(3)-X(1) )*(X(3)-X(1))-X(5)-H(8)*X(6))
   XDOT(7)=H(10)*(H(9)*X(6)*C-H(11)*X(7))
   XDOT(8)=H(12)*(H(9)*X(5)*C+X(7)-X(8))
 7 RETURN
   END
   END
```

```
              8         50         18
 0.0                  80.        0.005      0.0        0.0        0.0        0.0        0.0
 0.0                  0.0        0.0        0.0        0.0        0.0        0.0
13
 1                    9.
 2                    0.0
 3                    9.
 4                    9.
 5                    9.
 6                  .111
 7                  985.
 8                 .03185
 9                  4.
10                  0.02716
11                 0.1045
12                 .1
13                    30.
0          01.00E+01 04.00E+00          04   01   20   08             03
02 ERROR
02 POLE DRIFT
03 SINE WAVE INPUT,      SON
1    0 EROR
3    0 P.D
0          01.00E+01 04.00E+00          04   01   20   08             03
02 CORRECTION SIGNSL
02 POLE DRIFT
03 SINE WAVE INPUT,              SON
1    0 C.S
3    0 P.D
```

TIME	MODEL	PLANT	ERROR	CORSIG	POLSIG
.000	.00000E+00	.00000E+00	.00000E+00	.00000E+00	.00000E+00
.250	.78496E+00	.65736E+00	-.12760E+00	.35597E-01	.60356E+01
.500	.26836E+01	.22546E+01	-.42901E+00	.80853E-01	.52317E+01
.750	.38411E+01	.35108E+01	-.33027E+00	-.38405E+00	.36684E+01
1.000	.40636E+01	.39744E+01	-.89173E-01	-.36566E+00	.44477E+01
1.250	.34421E+01	.35493E+01	.10722E+00	-.43291E+00	.55669E+01
1.500	.20713E+01	.23557E+01	.28438E+00	-.36526E+00	.47988E+01
1.750	.24531E+00	.64320E+00	.39789E+00	-.66078E+00	.33968E+01
2.000	-.16333E+01	-.12369E+01	.39647E+00	-.89073E+00	.39046E+01
2.250	-.31558E+01	-.28449E+01	.31088E+00	-.11579E+01	.48414E+01
2.500	-.39910E+01	-.38131E+01	.17783E+00	-.13639E+01	.38135E+01
2.750	-.39577E+01	-.39432E+01	.14467E-01	-.12966E+01	.27664E+01
3.000	-.30625E+01	-.32066E+01	-.14408E+00	-.13352E+01	.34421E+01
3.250	-.14998E+01	-.17681E+01	-.26829E+00	-.13515E+01	.46470E+01
3.500	.38917E+00	.51772E-01	-.33740E+00	-.14774E+01	.37131E+01
3.750	.21934E+01	.18741E+01	-.31935E+00	-.17110E+01	.23575E+01
4.000	.35204E+01	.32972E+01	-.22322E+00	-.17812E+01	.29782E+01
4.250	.40801E+01	.39815E+01	-.98508E-01	-.18987E+01	.40989E+01
4.500	.37512E+01	.37897E+01	.38573E-01	-.18800E+01	.33238E+01
4.750	.26061E+01	.27777E+01	.17165E+00	-.18804E+01	.21939E+01
5.000	.89316E+00	.11495E+01	.25633E+00	-.20119E+01	.27297E+01
5.250	-.10144E+01	-.73698E+00	.27742E+00	-.21224E+01	.38739E+01
5.500	-.27007E+01	-.24559E+01	.24480E+00	-.23152E+01	.29017E+01
5.750	-.37993E+01	-.36401E+01	.15925E+00	-.23608E+01	.17194E+01
6.000	-.40708E+01	-.40252E+01	.45597E-01	-.23360E+01	.23879E+01
6.250	-.34550E+01	-.35242E+01	-.69210E-01	-.23203E+01	.36745E+01
6.500	-.20869E+01	-.22648E+01	-.17786E+00	-.22784E+01	.29517E+01
6.750	-.26484E+00	-.51085E+00	-.24601E+00	-.24013E+01	.16851E+01
7.000	.16153E+01	.13762E+01	-.23910E+00	-.25276E+01	.21787E+01
7.250	.31434E+01	.29605E+01	-.18291E+00	-.26517E+01	.33413E+01
7.500	.39867E+01	.38835E+01	-.10322E+00	-.27289E+01	.25142E+01
7.750	.39625E+01	.39601E+01	-.23619E-02	-.26758E+01	.14171E+01
8.000	.30754E+01	.31717E+01	.96302E-01	-.26633E+01	.20254E+01
8.250	.15180E+01	.16909E+01	.17297E+00	-.26638E+01	.33272E+01
8.500	-.36972E+00	-.15033E+00	.21939E+00	-.27329E+01	.25233E+01
8.750	-.21769E+01	-.19676E+01	.20928E+00	-.28556E+01	.12438E+01
9.000	-.35104E+01	-.33663E+01	.14413E+00	-.28980E+01	.17734E+01
9.250	-.40788E+01	-.40172E+01	.61639E-01	-.29286E+01	.30600E+01
9.500	-.37589E+01	-.37887E+01	-.29821E-01	-.28911E+01	.23781E+01
9.750	-.26210E+01	-.27431E+01	-.12207E+00	-.28671E+01	.12391E+01
10.000	-.91222E+00	-.10906E+01	-.17834E+00	-.29298E+01	.17242E+01
10.250	.99548E+00	.80536E+00	-.19011E+00	-.29963E+01	.29897E+01
10.500	.26860E+01	.25165E+01	-.16951E+00	-.31074E+01	.21748E+01
10.750	.37920E+01	.36806E+01	-.11141E+00	-.31278E+01	.98539E+00
11.000	.40725E+01	.40418E+01	-.30716E-01	-.30951E+01	.15418E+01
11.250	.34654E+01	.35162E+01	.50821E-01	-.30610E+01	.29220E+01
11.500	.21037E+01	.22350E+01	.13130E+00	-.30224E+01	.22727E+01
11.750	.28433E+00	.46815E+00	.18382E+00	-.30866E+01	.10337E+01
12.000	-.15973E+01	-.14202E+01	.17715E+00	-.31713E+01	.14486E+01
12.250	-.31309E+01	-.29970E+01	.13390E+00	-.32418E+01	.27381E+01